Economic Growth II:
Population Growth and Technological Progress

Macroeconomic Theory II

Plan

- The closed-economy Solow model with population growth
- How a country’s standard of living depends on its population rate
- Two simple models in which the rate of technological progress is endogenous

Population growth in Solow

Assume that the population and labor force grow at rate n (exogenous):

$$\frac{\Delta L}{L} = n$$

Example:

Suppose L = 1,000 in year 1 and the population is growing at 2% per year ($n = 0.02$).

Then $\Delta L = n\,L = 0.02 \times 1{,}000 = 20$, so L = 1,020 in year 2.

Break-even investment

- $(\delta + n)k$ = break-even investment,
  the amount of investment necessary to keep k constant

- Break-even investment includes:
  - δk to replace capital as it wears out
  - $n k$ to equip new workers with capital

- Otherwise, k would fall as the existing capital stock is spread more thinly over a larger population of workers.)

The equation of motion for k with population growth

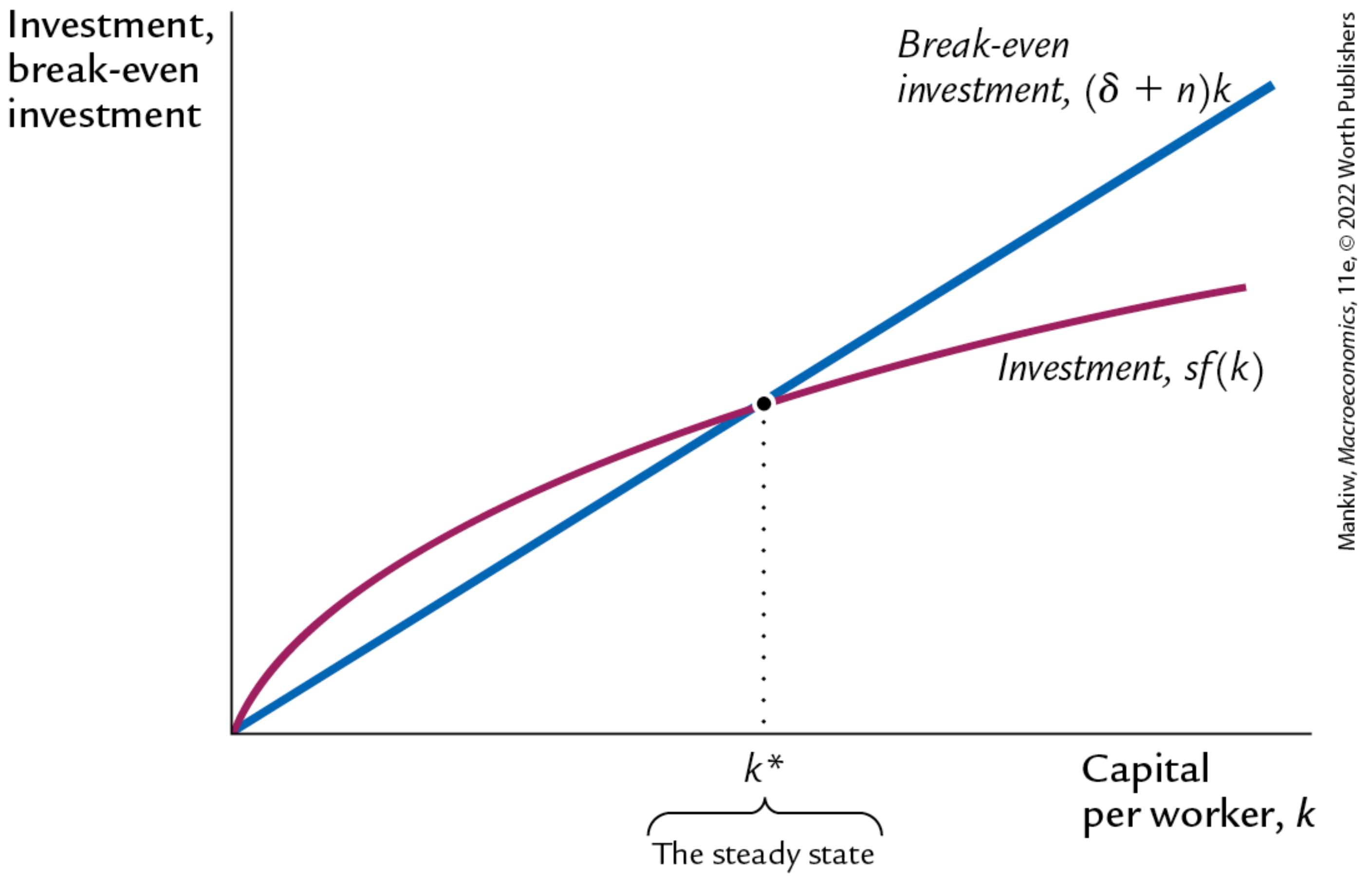
With population growth, the equation of motion for k is:

$$\Delta k = s\,f(k) - (\delta + n)\,k$$

$s\,f(k)$: actual investment

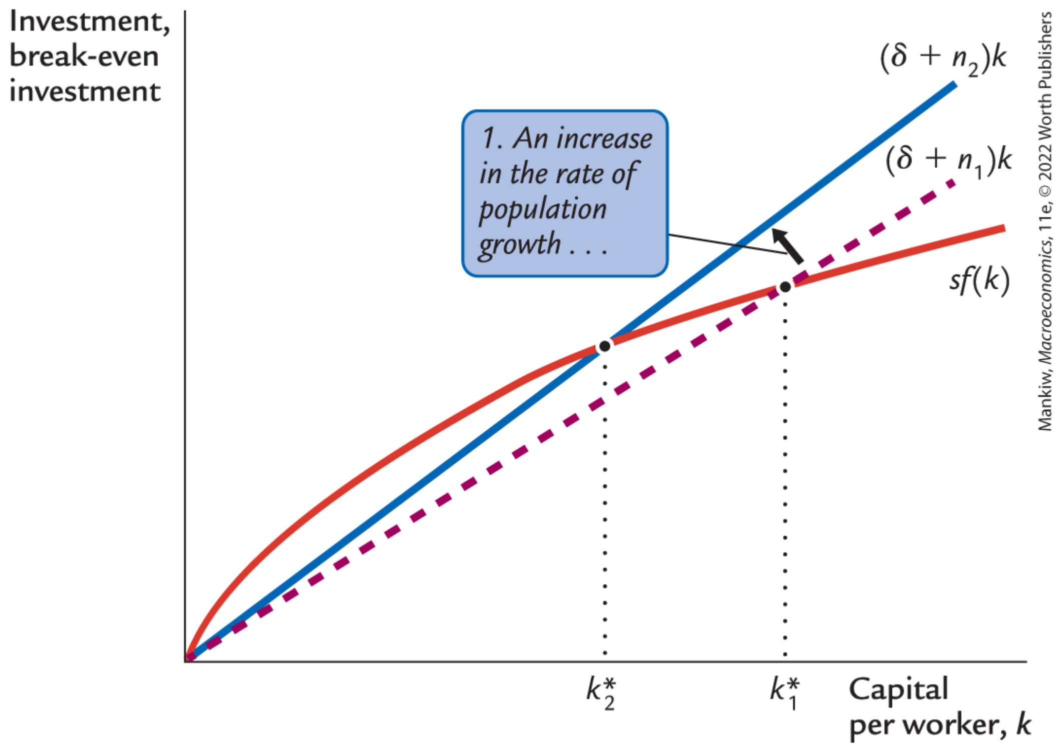
$(\delta + n)\,k$: break-even investment

The Solow model diagram



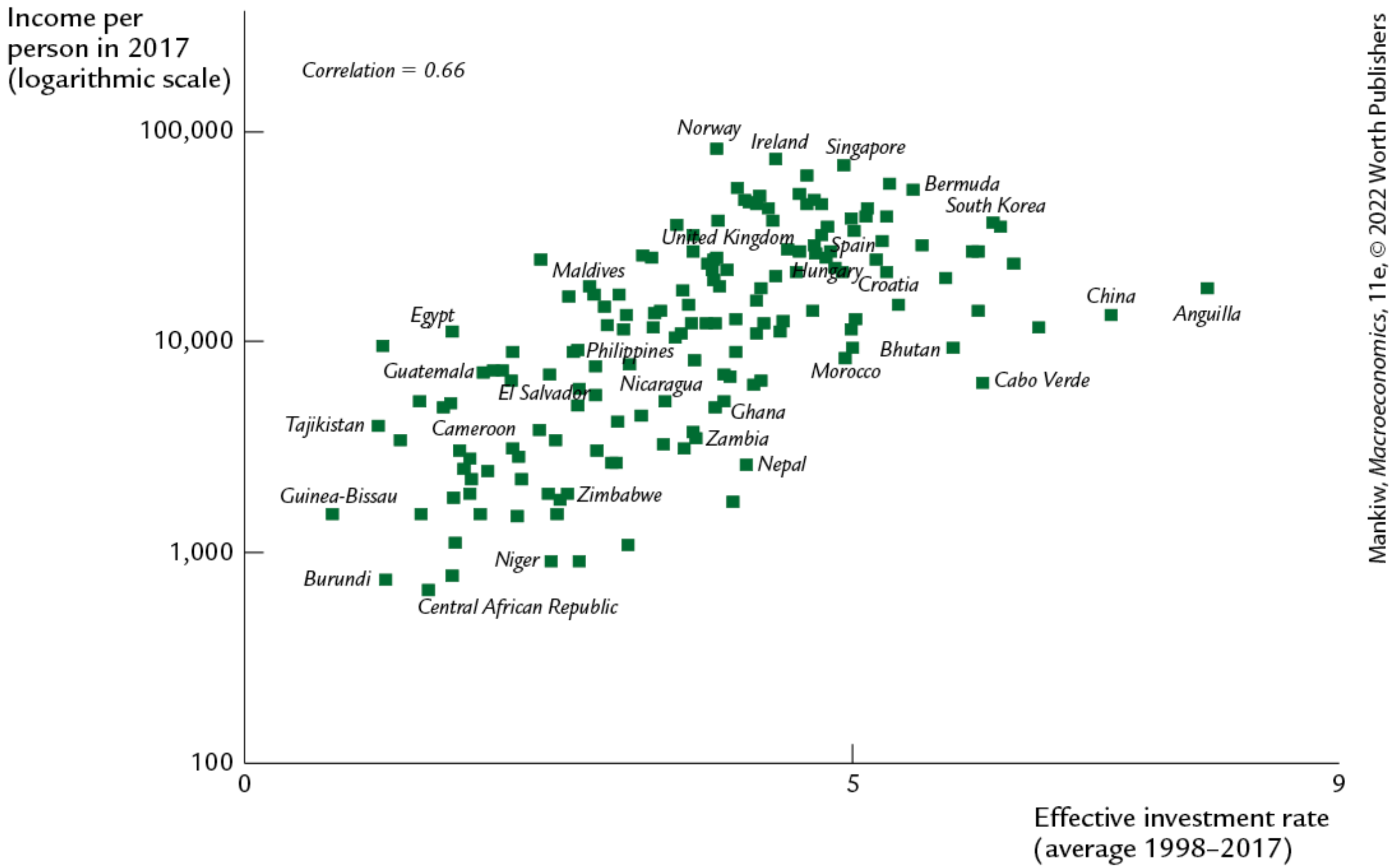
The impact of population growth

- An increase in ***n*** causes an increase in break-even investment, leading to a lower steady-state level of ***k***.



Prediction

- The Solow model predicts that countries with higher population growth rates will have lower levels of capital and income per worker in the long run.
- Are the data consistent with this prediction?


Income per person in 2017 (logarithmic scale)
Correlation = 0.66
100,000
10,000
1,000
100
0
5
9
Effective investment rate (average 1998–2017)
Norway
Ireland
Singapore
Bermuda
South Korea
United Kingdom
Spain
Hungary
Croatia
Maldives
China
Anguilla
Egypt
Philippines
Bhutan
Morocco
Cabo Verde
Guatemala
El Salvador
Nicaragua
Ghana
Tajikistan
Cameroon
Zambia
Nepal
Guinea-Bissau
Zimbabwe
Niger
Burundi
Central African Republic
Mankiw, Macroeconomics, 11e, © 2022 Worth Publishers

The Golden Rule with population growth

- To find the Golden Rule capital stock, express $\boldsymbol{c}^*$ in terms of $\boldsymbol{k}^*$:

$$c^* = y^* - i^*$$

$$= f(k^*) - (\delta + n)\,k^*$$

c^* is maximized when

$$\text{MPK} = \delta + n$$

Or, equivalently,

$$\text{MPK} - \delta = n$$

In the Golden Rule steady state, the marginal product of capital net of depreciation equals the population growth rate.

Alternative perspectives on population growth

The Malthusian model (1798)

- It predicts population growth will outstrip the Earth's ability to produce food, leading to the impoverishment of humanity.
- Since the time of Malthus, world population has increased sixfold, yet living standards are higher than ever.
- Malthus neglected the effects of technological progress.

Alternative perspectives on population growth

The Kremerian model (1993)

- Posits that population growth contributes to economic growth.
- More people = more geniuses, scientists, and engineers, so faster technological progress.
- Evidence from very long historical periods shows that:
  - as world population growth rate increased, so did the rate of growth in living standards.
  - historically, regions with larger populations have enjoyed faster growth.

Adding technological progress into the Solow Model

- In the Solow model,
  - the production technology is held constant.
  - income per capita is constant in the steady state.

- We introduce a new variable: E = labor efficiency

- Assume:
  Technological progress is labor-augmenting:
  it increases labor efficiency at the exogenous rate g:

$$g = \frac{\Delta E}{E}$$

Technological progress in the Solow model

- We now write the production function as:

$$Y = F(K, L \times E)$$

  - where $L \times E$ = number of effective workers
    - Increases in labor efficiency have the same effect on output as increases in the labor force.

Technological progress in the Solow model

- Notation:

  $\boldsymbol{y} = \boldsymbol{Y / LE}$ = output per effective worker
  $\boldsymbol{k} = \boldsymbol{K / LE}$ = capital per effective worker

- Production function per effective worker:
  $\boldsymbol{y = f(k)}$

- Saving and investment per effective worker:
  $\boldsymbol{s\,y = s\,f(k)}$

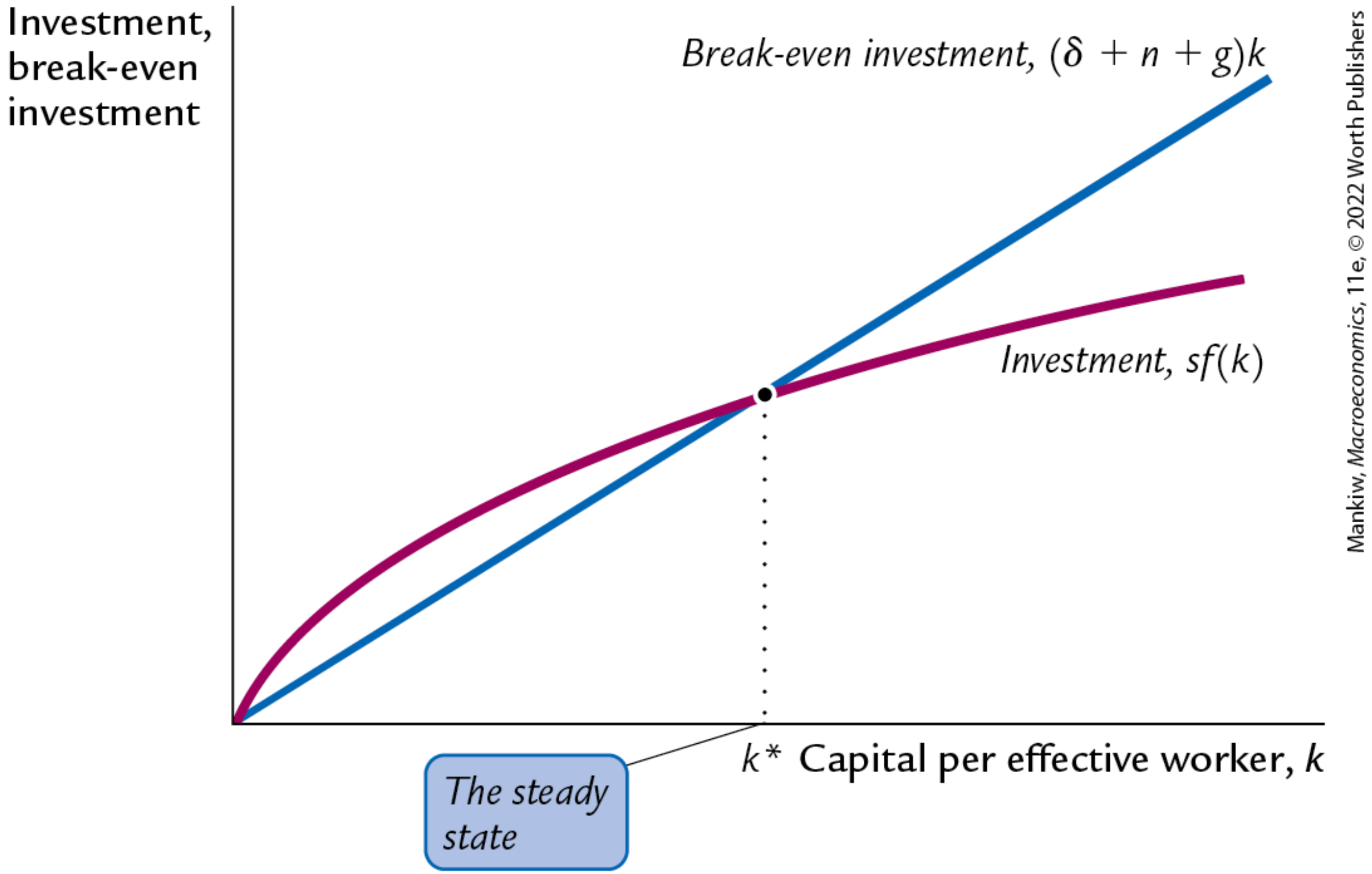
Technological progress in the Solow model

- $(\boldsymbol{\delta} + \boldsymbol{n} + \boldsymbol{g})\, \boldsymbol{k}$ = break-even investment:
  the amount of investment necessary to keep ***k*** constant.

- Consists of:
  - $\boldsymbol{\delta}\, \boldsymbol{k}$ to replace depreciating capital
  - $\boldsymbol{n}\, \boldsymbol{k}$ to provide capital for new workers
  - $\boldsymbol{g}\, \boldsymbol{k}$ to provide capital for the new "effective" workers created by technological progress

Technological progress in the Solow model

$$\Delta k = s\,f(k) - (\delta + n + g)k$$



Steady-state growth rates in the Solow model with technological progress

Variable	Symbol	Steady-State Growth Rate
Capital per effective worker	$k = K/(E \times L)$	0
Output per effective worker	$y = Y/(E \times L) = f(k)$	0
Output per worker	$Y/L = y \times E$	g
Total output	$Y = y \times (E \times L)$	$n + g$

The Golden Rule with technological progress

To find the Golden Rule capital stock, express c^* in terms of k^*:

$$c^* = y^* - i^*$$

$$= f(k^*) - (\delta + n + g)k^*$$

c^* is maximized when

$$MPK = \delta + n + g$$

Or, equivalently,

$$MPK - \delta = n + g$$

In the Golden Rule steady state, the marginal product of capital net of depreciation equals the population growth rate plus the rate of tech progress.

Endogenous growth theory

Solow model:

- Sustained growth in living standards is due to technological progress.
- The rate of technological progress is exogenous.

Endogenous growth theory:

- In this set of models, the growth rate of productivity and living standards is endogenous.

Endogenous growth theory

- Production function: $Y = A K$
  where A is the amount of output for each unit of capital (A is exogenous and constant)

- Key difference between this model and Solow:
  MPK is constant here, diminishes in Solow

- Investment: sY
- Depreciation: δK
- Equation of motion for total capital: $\Delta K = sY - \delta K$

Endogenous growth theory

- $\Delta \boldsymbol{K} = \boldsymbol{sY} - \boldsymbol{\delta K}$
- Divide through by $\boldsymbol{K}$ and use $\boldsymbol{Y} = \boldsymbol{A}\,\boldsymbol{K}$ to get:

$$\frac{\Delta Y}{Y} = \frac{\Delta K}{K} = sA - \delta$$

- If $s\,A > \delta$, then income will grow forever, and investment is the "engine of growth."
- Here, the permanent growth rate depends on s. In Solow model, it does not.

Does capital have diminishing returns or not?

- It depends on the definition of capital.
- If capital is narrowly defined (only plant and equipment), then yes.
- Advocates of endogenous growth theory argue that knowledge is a type of capital.
- If so, then constant return to capital is more plausible, and this model may be a good description of economic growth.

A two-sector model, part 1

- Two sectors:
  - Manufacturing firms produce goods.
  - Research universities produce knowledge that increases labor efficiency in manufacturing.
- $\boldsymbol{u}$ = fraction of labor in research ($\boldsymbol{u}$ is exogenous)
- Manufacturing production function:
  - $\boldsymbol{Y} = \boldsymbol{F}\,[\boldsymbol{K}, (1 - \boldsymbol{u})\boldsymbol{E}\,\boldsymbol{L}]$
- Research production function: $\Delta \boldsymbol{E} = \boldsymbol{g}\,(\boldsymbol{u})\boldsymbol{E}$
- Capital accumulation: $\Delta \boldsymbol{K} = \boldsymbol{s}\,\boldsymbol{Y} - \boldsymbol{\delta K}$

A two-sector model, part 2

- In the steady state, manufacturing output per worker and the standard of living grow at rate:

$$\Delta E / E = g(u)$$

- Key variables:

  s: affects the level of income but not its growth rate
  (same as in the Solow model)

  u: affects level and growth rate of income

DISCUSSION QUESTION: The merits of raising *u*

Question:

- In what ways would raising ***u*** (that is, devoting more labor to research) benefit the economy?
- What are the costs of raising ***u***?

Facts about R&D

- Much research is done by firms seeking profits.
- Firms profit from research:
  - Patents create a stream of monopoly profits.
  - There is extra profit in being first on the market with a new product.
- Innovation produces externalities that reduce the cost of subsequent innovation.

* Much of the new endogenous growth theory attempts to incorporate these facts into models to better understand technological progress.

Is the private sector doing enough R&D?

- The existence of positive externalities in the creation of knowledge suggests that the private sector is not doing enough R&D.
- But there is much duplication of R&D effort among competing firms.
- Estimates:
  social return to R&D $\geq$ 40 percent per year
- Thus, many believe the government should encourage R&D.

Economic growth as "creative destruction"

- Schumpeter (1942) coined term "creative destruction" to describe displacements resulting from technological progress:
- The introduction of a new product is good for consumers but often bad for incumbent producers, who may be forced out of the market.

Examples:

- Luddites (1811–1812) destroyed machines that displaced skilled mill workers in England.
- Walmart displaces many mom-and-pop stores.

Summary

- Key results from the Solow model with technological progress:
  - The steady-state growth rate of income per person depends solely on the exogenous rate of technological progress.
- Endogenous growth theory: Models that
  - examine the determinants of the rate of technological progress, which Solow takes as given.
  - explain decisions that determine the creation of knowledge through R&D.

Reference

- Mankiw's Macroeconomics – Chapter 9