Growth Theories

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Macroeconomic Theory

The Solow Model I

- A seminal model of economic growth is the Solow growth model.
- We review the useful insights from this model and also discuss its limitations.
- The Solow model shows that capital accumulation cannot sustain long-run economic growth, which requires technological progress.
- However, the model treats technological progress as exogenous and does not inform us on its determinants.
- The Solow model consists of the following components:
  - an aggregate production function,
  - an accumulation equation for capital,
  - an exogenous saving rate.

The Solow Model without Technological Progress I

Output Y_t at time t is produced by the Cobb-Douglas production function:

$$Y_t = AK_t^{\alpha}L_t^{1-\alpha}$$

- The parameter $\alpha \in (0,1)$ is the degree of capital intensity in production.
- A is the exogenous level of technology.
- K_t is the stock of capital that has been accumulated as of time t.
- L_t is the size of the labour force in the economy at time t.

For simplicity, we assume that the size of the labour force is constant, and we normalise L_t to unity so that other variables can be interpreted as per capita variables (e.g., output per capita).

The Solow Model without Technological Progress II

The second key equation in this model is the accumulation equation for capital given by

$$\dot{K}_t = I_t - \delta K_t$$

- The parameter $\delta > 0$ is the depreciation rate of capital,
- $\dot{K}_t \equiv \partial K_t / \partial t$ denotes the change in the stock of capital with respect to time t.
- I_t is capital investment.

The Solow Model without Technological Progress III

In this closed economy without a government sector, the national income account is simply

$$Y_t = C_t + I_t$$

where C_t is consumption in the economy at time t.

The Solow growth model is quite simple because it assumes an exogenous saving (or investment) rate denoted by s. In other words,

$$s \equiv \frac{I_t}{Y_t} = 1 - \frac{C_t}{Y_t}.$$

The Solow Model without Technological Progress IV

To solve this model, we substitute Cobb-Douglas production function and exogenous saving equation into capital accumulation equation to obtain

$$\dot{K}_t = sAK_t^{\alpha} - \delta K_t$$

where we have used $L_t = 1$. This equation is a one-dimensional differential equation in K_t. Imposing $\dot{K}_t = 0$ yields the steady-state level of capital given by

$$K^* = \left(\frac{sA}{\delta}\right)^{1/(1-\alpha)}$$

The Solow Model without Technological Progress V

$$\dot{K}_t = sAK_t^{\alpha} - \delta K_t$$

Equation implies :
- whenever $K_t < K^*, K_t$ would increase over time until it reaches K^* (see Figure 9.1).
- whenever, $K_t > K^*, K_t$ would decrease over time until it reaches K^*.

$$K^* = \left(\frac{sA}{\delta}\right)^{1/(1-\alpha)}$$

Equation shows that K^* is increasing in the saving rate s and the level of technology A but decreasing in the depreciation rate δ.

The Solow Model without Technological Progress VI

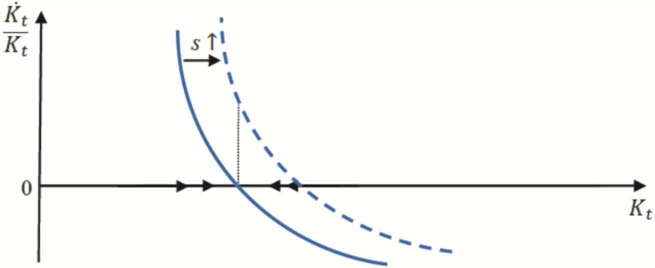

Figure 9.1. Phase diagram.

The Solow Model without Technological Progress VII

$\Rightarrow$ When the level of capital converges to its steady state K^*, the level of output also reaches its steady-state level given by $Y^* = A(K^*)^\alpha$.

$\Rightarrow$ Although Y^* is increasing in the saving rate s, the level of output is stationary in the long run.

Why doesn't the output keep growing in the long run?

To answer this question, we rewrite

$$\dot{K}_t = sAK_t^\alpha - \delta K_t$$

to derive an expression for the growth rate of capital.

$$\frac{\dot{K}_t}{K_t} = \frac{sA}{K_t^{1-\alpha}} - \delta$$

The Solow Model without Technological Progress VIII

$$\frac{\dot{K}_t}{K_t} = \frac{sA}{K_t^{1-\alpha}} - \delta$$

- This equation shows that as K_t increases, the growth rate of capital $\dot{K}_t/K_t$ decreases and eventually converges to a long-run value of zero.
- An increase in the saving rate s would increase the growth rate of capital in the short run, but the growth rate of capital always converges to zero in the long run (see Figure 9.1).
- The reason behind this convergence process is decreasing returns to scale (i.e., $\alpha < 1$) with respect to capital in the production function.
- As capital increases, output increases; however, the additional output that the additional capital produces is decreasing.

The Solow Model without Technological Progress IX

- This diminishing marginal product of capital implies that the additional investment created by the additional output is also decreasing.
- Given that capital accumulation requires capital investment, the growth rate of capital decreases and converges to zero.
- If the production function instead features constant returns to scale (i.e., $\alpha = 1$), then the long-run growth rate of capital would be $\dot{K}_t / K_t = sA - \delta$, which remains positive so long as $sA > \delta$.

The Solow Model with Technological Progress I

⇒ The previous section shows that in the more plausible case of decreasing returns to scale (i.e., $\alpha < 1$), the stock of capital would converge to a steady state without economic growth in the long run.

⇒ This result arises because there is no technological progress (i.e., A is assumed to be a constant parameter).

⇒ In the rest of this section, we analyse the more interesting case in which A_t is a variable that grows over time according to an exogenous growth rate $g_A \equiv \dot{A}_t / A_t > 0$.

Taking the natural log of the production function $Y_t = A_t K_t^{\alpha}$ yields

$$\ln Y_t = \ln A_t + \alpha \ln K_t.$$

The Solow Model with Technological Progress II

Differentiating this equation with respect to t yields

$$\frac{\dot{Y}_t}{Y_t} = \frac{\dot{A}_t}{A_t} + \alpha \frac{\dot{K}_t}{K_t},$$

where $\dot{A}_t / A_t = g_A$ is an exogenous parameter.

* Recall that $\frac{\partial \ln Y_t}{\partial t} = \frac{1}{Y_t} \frac{\partial Y_t}{\partial t} = \frac{\dot{Y}_t}{Y_t}$.

Substituting saving rate equation into capital equation and dividing by K_t yields

$$\frac{\dot{K}_t}{K_t} = s \frac{Y_t}{K_t} - \delta.$$

The Solow Model with Technological Progress III

$$\frac{\dot{K}_t}{K_t} = s\frac{Y_t}{K_t} - \delta.$$

- In the long run, the economy is on a balanced growth path (BGP) along which each variable grows at a constant rate.
- Because this constant growth rate can be zero, a steady state is a special case of a balanced growth path.
- Because $\dot{K}_t/K_t$ is constant on the BGP, Y_t/K_t is also constant implying that output Y_t and capital K_t grow at the same rate on the BGP.

The Solow Model with Technological Progress IV

Using this information and

$$\frac{\dot{Y}_t}{Y_t} = \frac{\dot{A}_t}{A_t} + \alpha \frac{\dot{K}_t}{K_t},$$

we can now derive the long-run growth rate of output Y_t and capital K_t on the BGP:

$$\frac{\dot{Y}_t}{Y_t} = \frac{\dot{K}_t}{K_t} = \frac{g_A}{1-\alpha}$$

*** This equation reveals a key insight of the Solow model: economic growth in the long run is driven by technological progress.

The Solow Model with Technological Progress V

- Where does technological progress come from?
- Unfortunately, the Solow model cannot be used to analyse this question because technological progress is exogenous in the model.
- In other words, although the Solow model allows us to explore the relative importance of technological progress and capital accumulation, which Acemoglu (2009) refers to as proximate causes of economic growth, it does not allow us to explore the fundamental causes of economic growth.
- The Romer model in which the technology growth rate g_A is endogenously determined by R&D and innovation in the market economy, which allows us to explore how fundamental causes, such as culture (the household's preference) and institutions (intellectual property rights), affect economic growth via technological progress.

The Solow Model with Technological Progress VI

- Similarly, the Solow model features an exogenous saving rate, which determines capital accumulation as a proximate cause of growth.
- However, it cannot be used to analyse the question on why the saving rate differs across countries.

Summary I

- We review the Solow growth model.
- In the absence of technological progress, the economy always converges to a steady state, in which the long-run levels of capital and output are stationary and increasing in the saving rate and the level of technology but decreasing in the depreciation rate of capital.
- The absence of long-run economic growth is due to the decreasing returns to scale with respect to capital in the production function.
- Therefore, unless capital exhibits constant returns to scale in production, capital accumulation alone cannot sustain economic growth in the long run without technological progress.
- In the presence of technological progress, the long-run growth rate of output and capital is determined by the growth rate of technology, which however is exogenous in the Solow growth model.

Summary II

- Therefore, one cannot use the Solow growth model to explore the determinants of technological progress.

The Basics I

- The Ramsey model can be viewed as a generalisation of the Solow growth model.
- The Solow model assumes an exogenous saving rate, whereas the Ramsey model features a representative household which chooses the saving rate optimally.
- As we saw in the Solow model, although the saving rate does not affect the long-run growth rate, it affects the levels of capital and output.
- Therefore, the Ramsey model allows us to explore the question on why the saving rate differs across countries, which explains some of the variation in the level of income across countries.

The Basics II

- In summary, we find that the saving rate is determined by the household's discount rate, the degree of capital intensity in production and the depreciation rate of capital but independent of the level of technology.
- Aside from the endogenous saving rate, the rest of the Ramsey model is the same as the Solow model.

Household I

In the Ramsey model, there is a representative household, which has a utility function u_t at time t.
As in the neoclassical growth model, we consider a log utility function:

$$u_t = \ln C_t$$

which depends on consumption C_t at time t.

Given the discount rate $\rho > 0$, the lifetime utility function is given by

$$U = u_0 + \frac{u_1}{1+\rho} + \frac{u_2}{(1+\rho)^2} + \cdots = \sum_{t=0}^{\infty} \frac{u_t}{(1+\rho)^t}$$

where we assume that a lifetime is long enough to be approximated by infinity.
* As t becomes very large, the discounting would make $u_t/(1+\rho)^t$ not to matter too much in the utility function U.

Household II

In this analysis, we will once again use the Hamiltonian to solve the household's dynamic optimisation problem in continuous time.

Therefore, we need to rewrite utility function in continuous time using the integral as

$$U = \int_0^\infty e^{-\rho t} u_t dt = \int_0^\infty e^{-\rho t} \ln C_t dt$$

where the continuous-time discount factor $e^{-\rho t}$ replaces the discrete-time discount factor $(1+\rho)^{-t}$.

As in the Solow model, output Y_t at time t is produced by the Cobb-Douglas production function:

$$Y_t = AK_t^{\alpha} L_t^{1-\alpha},$$

Household III

$$Y_t = AK_t^{\alpha} L_t^{1-\alpha},$$

- the parameter $\alpha \in (0, 1)$ is the degree of capital intensity in production
- A is the exogenous level of technology
- K_t is the stock of capital that has been accumulated as of time t
- L_t is the size of the labour force, which we normalise to unity.

The accumulation equation for capital is

$$\dot{K}_t = I_t - \delta K_t$$

where the parameter $\delta > 0$ is the depreciation rate of capital, and I_t is capital investment.

Household IV

In this closed economy without a government sector, the national income account is simply

$$Y_t = C_t + I_t.$$

Substituting C-D production function and national income equation into capital accumulation yields

$$\dot{K}_t = Y_t - C_t - \delta K_t = AK_t^{\alpha} - C_t - \delta K_t$$

where we have set $L_t = 1$.

With the above information, we can now set up the dynamic optimisation problem faced by the representative household.

Household V

The household chooses consumption C_t and accumulates capital K_t in order to maximise lifetime utility.

Formally, the optimisation problem is

$$\max_{C_t} U = \int_0^\infty e^{-\rho t} \ln C_t dt$$

subject to

$$\dot{K}_t = AK_t^\alpha - C_t - \delta K_t$$

To solve the dynamic optimisation problem, we use the Hamiltonian.

Hamiltonian I

- First, we set up the Hamiltonian function.
- Then, we derive the first-order conditions.
- Finally, we use the first-order conditions to derive the steady-state levels of C_t and K_t.

The Hamiltonian function is given by

$$H_t = \ln C_t + \lambda_t \left(AK_t^{\alpha} - C_t - \delta K_t \right)$$

The Hamiltonian consists of
(a) the utility function $\ln C_t$ at time t,
(b) the capital-accumulation equation $AK_t^{\alpha} - C_t - \delta K_t$,
(c) a multiplier λ_t for the capital-accumulation equation.

Hamiltonian II

Now, we derive the first-order conditions with respect to C_t and K_t. The first-order conditions include

$$\frac{\partial H_t}{\partial C_t} = \frac{1}{C_t} - \lambda_t = 0$$
$$\frac{\partial H_t}{\partial K_t} = \lambda_t \left(\alpha A K_t^{\alpha-1} - \delta\right) = \lambda_t \rho - \dot{\lambda}_t$$

Note that K_t is a state variable (i.e., a variable that accumulates over time), so we have to set $\partial H_t / \partial K_t = \lambda_t \rho - \dot{\lambda}_t$.
From FOC-1, we have $C_t = \lambda_t^{-1}$.
Taking the log of this equation yields

$$\ln C_t = -\ln \lambda_t$$

Hamiltonian III

Differentiating it with respect to t yields

$$\frac{\dot{C}_t}{C_t} = -\frac{\dot{\lambda}_t}{\lambda_t}$$

Substituting this equation into FOC-2 yields

$$\frac{\dot{C}_t}{C_t} = -\frac{\dot{\lambda}_t}{\lambda_t} = \alpha A K_t^{\alpha-1} - \delta - \rho$$

- Final equation is the optimal path of consumption chosen by the household.
- The optimal path of consumption states that if the net return to capital (i.e., the marginal product of capital $\alpha A K_t^{\alpha-1}$ net of depreciation δ) is greater than the discount rate ρ, then the household should save more and consume less today.

Hamiltonian IV

- Because current consumption is relatively low, consumption must be increasing over time so that $\dot{C}_t > 0$.
- On the other hand, if the net return to capital is less than the discount rate, then the household should save less and consume more today.
- Because current consumption is relatively high, consumption must be decreasing over time so that $\dot{C}_t < 0$.

Steady State I

In summary, the Ramsey model provides us with two differential equations:

$$\frac{\dot{C}_t}{C_t} = \alpha A K_t^{\alpha-1} - \delta - \rho$$
$$\dot{K}_t = A K_t^{\alpha} - C_t - \delta K_t$$

Now, we solve for the steady state.
In the steady state, $\dot{C}_t = 0$ and $\dot{K}_t = 0$. Imposing $\dot{C}_t = 0$ on the optimal consumption path yields the steady-state level of capital:

$$K^* = \left(\frac{\alpha A}{\rho + \delta}\right)^{1/(1-\alpha)}$$

- increasing in the level of technology A and decreasing in the discount rate ρ and the depreciation rate δ.

Steady State II

- Intuitively, a higher level of technology A increases the return to capital and encourages the household to accumulate more capital.
- In contrast, a higher discount rate ρ makes future consumption less attractive to the household, which then accumulates less capital.
- Finally, a higher depreciation rate δ also makes capital depreciate more rapidly, so that the level of accumulated capital becomes lower.

Imposing $\dot{K}_t = 0$ on the capital-accumulation equation yields the steady-state level of consumption:

$$C^* = A\left(K^*\right)^{\alpha} - \delta K^* = \frac{\rho + \delta(1-\alpha)}{\rho + \delta}\left(\frac{\alpha A}{\rho + \delta}\right)^{\alpha/(1-\alpha)} A$$

Steady State III

Using the production function, we can also derive the steady-state level of output given by

$$Y^* = A(K^*)^\alpha = \left(\frac{\alpha A}{\rho+\delta}\right)^{\alpha/(1-\alpha)} A$$

As for the steady-state level of investment, it is given by

$$I^* = Y^* - C^* = \delta K^* = \left(\frac{\alpha A}{\rho+\delta}\right)^{1/(1-\alpha)} \delta$$

To derive the steady-state saving rate s^*, we use

$$s^* \equiv \frac{I^*}{Y^*} = \frac{\delta K^*}{A(K^*)^\alpha} = \frac{\alpha\delta}{\rho+\delta},$$

which is increasing in the degree of capital intensity α and the depreciation rate δ but decreasing in the discount rate ρ.

Steady State IV

The intuition of these results can be explained as follows.

- If capital becomes more important in production (i.e., a larger α), then the household would save more to accumulate capital.
- A higher discount rate ρ makes future consumption less attractive to the household, which then saves less.
- As for the capital depreciation rate δ, it has two effects on the household's saving rate.
- First, it reduces the net return to capital and makes saving less attractive, which is captured by the δ in the denominator of s^*.
- Second, capital depreciation requires investment to replace the depreciated capital.
- This replacement effect, which is captured by the δ in the numerator of s^*, implies that a larger δ requires a higher saving rate s^*.

Steady State V

- Overall, the positive replacement effect of δ dominates unless $\rho \to 0$, in which case the two effects exactly offset each other.
- Finally, we substitute $s^*/\delta = \alpha/(\rho + \delta)$ into steady-state level of capital to obtain.

$$K^* = \left(\frac{s^* A}{\delta}\right)^{1/(1-\alpha)}$$

which is the same as K^* in Solow Model except that s is exogenous in the Solow model whereas s^* is endogenous in the Ramsey model.

Summary I

- The Ramsey model generalises the Solow growth model by featuring an endogenous saving rate that is optimally chosen by a utility-maximising household.
- The Solow growth model shows that an increase in the exogenous saving rate gives rise to a higher growth rate in the short run and a higher level of output in the long run, which demonstrates the importance of capital accumulation as a proximate cause of economic growth.
- However, one cannot use the Solow growth model to explore the fundamental determinants of capital accumulation, which the Ramsey model allows us to do.

Summary II

- In summary, we find that the steady-state saving rate is increasing in the degree of capital intensity and the depreciation rate of capital but decreasing in the discount rate, which reflects the preference of the representative household and captures a cultural trait.
- Therefore, cross-country variation in these determinants helps to explain some of the variation in income level across countries.

Intro I

- In the Ramsey model, the representative household carries out the production of goods.
- However, this setting is unrealistic because goods are often produced by firms in the real world.
- Now, we introduce a market economy to the Ramsey model, in which the representative household supplies labour and capital to a representative firm, which then uses these factor inputs to produce output and sells the output back to the household.
- As you can see, this familiar setting is basically the neoclassical growth model.
- After deriving the equilibrium allocation of resources in the decentralised market economy, we can then compare it to the allocation in the centralised economy that is optimally chosen by the representative household.

Intro II

- In summary, we find that the two sets of allocations are the same, implying that the market economy is efficient.
- In other words, the first fundamental theorem of welfare economics holds in this setting due to the absence of distortion in the market economy.

Household I

In the Ramsey model, there is a representative household, which has the following lifetime utility function:

$$U = \int_0^{\infty} e^{-\rho t} \ln C_t dt$$

- The parameter $\rho > 0$ is the household's discount rate and C_t is the level of consumption at time t.
- The household inelastically supplies one unit of labour to earn a wage income W_t.
- Furthermore, it accumulates capital K_t and rents it to the representative firm to earn a capital-rental income R_t.

Household II

If we normalise the price of output to unity, then the asset-accumulation equation:

$$\dot{K}_t = R_t K_t + W_t - C_t - \delta K_t$$

where the parameter $\delta > 0$ is the depreciation rate of capital.

To solve this dynamic optimisation problem, we use the Hamiltonian. The Hamiltonian function is given by

$$H_t = \ln C_t + \lambda_t \left(R_t K_t + W_t - C_t - \delta K_t \right)$$

The first-order conditions include

$$\frac{\partial H_t}{\partial C_t} = \frac{1}{C_t} - \lambda_t = 0$$

$$\frac{\partial H_t}{\partial K_t} = \lambda_t \left(R_t - \delta \right) = \lambda_t \rho - \dot{\lambda}_t$$

Household III

Recall once again that K_t is a state variable (i.e., a variable that accumulates over time).

Taking the log of FOC-1:

$$\ln C_t = -\ln \lambda_t$$

Differentiating it with respect to t yields

$$\frac{\dot{C}_t}{C_t} = -\frac{\dot{\lambda}_t}{\lambda_t}$$

Substituting this equation into FOC-2 yields

$$\frac{\dot{C}_t}{C_t} = R_t - \delta - \rho$$

which is the optimal path of consumption chosen by the household.

Firm I

There is a representative firm in the economy, and this firm hires labour L_t and rents capital K_t from the household to produce output Y_t using the following production function:

$$Y_t = AK_t^{\alpha}L_t^{1-\alpha}$$

where the parameter $\alpha \in (0, 1)$ is the degree of capital intensity in production and A is the exogenous level of technology.

The profit function Π_t is

$$\Pi_t = Y_t - R_tK_t - W_tL_t$$

where we have chosen Y_t as the numeraire (i.e., the price of Y_t is normalised to unity).

Firm II

Differentiating profit function with respect to K_t and L_t yields

$$\frac{\partial \Pi_t}{\partial K_t} = \frac{\partial Y_t}{\partial K_t} - R_t = \alpha A K_t^{\alpha-1} L_t^{1-\alpha} - R_t = 0$$
$$\frac{\partial \Pi_t}{\partial L_t} = \frac{\partial Y_t}{\partial L_t} - W_t = (1-\alpha) A K_t^{\alpha} L_t^{-\alpha} - W_t = 0$$

These two equations are the demand functions for K_t and L_t.

Equilibrium I

Substituting demand function for capital into the consumption path yields

$$\frac{\dot{C}_t}{C_t} = \alpha A K_t^{\alpha-1} - \delta - \rho,$$

where we have set $L_t = 1$.

Substituting the demand functions for K_t and L_t into asset-accumulation equation yields the capital-accumulation equation:

$$\dot{K}_t = \alpha A K_t^{\alpha} + (1-\alpha) A K_t^{\alpha} - C_t - \delta K_t = A K_t^{\alpha} - C_t - \delta K_t$$

Equations above are two differential equations in C_t and K_t, and these two equations completely characterise the behaviour of the economy.

Equilibrium II

- It is important to note that two differential equations in C_t and K_t are exactly the same as in the Ramsey model.
- Therefore, the decentralised market economy has the same allocation of resources as the centralised economy that is optimally chosen by the representative household.
- The reason is that the rental price R_t in the market economy is equal to the marginal product of capital $\alpha A K_t^{\alpha-1}$.
- This also implies that the decentralised economy has the same steady-state equilibrium allocations as the centralised economy.

Equilibrium III

For example, the steady-state equilibrium level of capital is the same as before such that

$$K^* = \left(\frac{\alpha A}{\rho + \delta}\right)^{1/(1-\alpha)}$$

and the household's saving rate s^* is also the same as before such that

$$s^* \equiv \frac{I^*}{Y^*} = \frac{\delta K^*}{A\left(K^*\right)^{\alpha}} = \frac{\alpha\delta}{\rho + \delta}.$$

This result is an example of the first fundamental theorem of welfare economics, which states that any competitive equilibrium leads to a Pareto efficient allocation of resources.

This fundamental theorem holds because the decentralised economy is characterised by perfect competition in the production sector, so that $R_t = MPK_t = \alpha A K_t^{\alpha-1}$.

Equilibrium IV

Next , we will consider another version of the Ramsey model with monopolistic competition under which the decentralised economy differs from the centralised economy (i.e., the decentralised economy exhibits market failure).

Summary I

- We introduce a market economy into the Ramsey model.
- Specifically, we consider a perfectly competitive product market. In this case, a representative firm demands factor inputs from the representative household and supplies output to the household.
- Therefore, the household and the firm interact in the labour market, the capital market and the product market.
- We find that the equilibrium allocation of resources in the market economy is the same as the socially optimal allocation in the centralised version of the Ramsey model.
- In other words, due to the absence of distortion in the economy, the market equilibrium is efficient.
- Therefore, the first fundamental theorem of welfare economics holds in the Ramsey model with a perfectly competitive product market.

Intro I

- We will consider another form of market structure in the Ramsey model.
- Specifically, we consider monopolistic competition.
- In the model, there is a representative firm that produces the final good. Also, there are a number of firms that produce differentiated intermediate goods.
- Because each of these intermediate-good firms sells a differentiated product, it has market power and charges a markup over the marginal cost.
- This monopolistic distortion in turn causes the decentralised economy to have a different allocation of resources from the centralised economy.

Intro II

- In other words, the first fundamental theorem of welfare economics does not hold in this setting due to the distortion of monopolistic competition, which in turn gives rise to market failure.

The model has the following components:

- (a) a representative household,
- (b) a representative firm that produces the final good,
- (c) N monopolistic firms that produce differentiated intermediate goods.

Household I

The representative household has the following lifetime utility function:

$$U = \int_0^\infty e^{-\rho t} \ln C_t dt$$

- the parameter $\rho > 0$ is the household's discount rate
- C_t is the level of consumption at time t
- the household inelastically supplies one unit of labour to earn a wage income W_t
- Furthermore, it accumulates capital K_t and rents it to firms to earn a capital-rental income R_t.

Household II

The asset-accumulation equation:

$$\dot{K}_t = R_t K_t + W_t + \sum_{i=1}^{N} \pi_t(i) - C_t - \delta K_t$$

- the parameter $\delta > 0$ is the depreciation rate of capital.
- It is useful to note that each intermediate-good firm i generates monopolistic profit $\pi_t(i)$, which is transferred back to the household as it owns the firm.

!! We have dealt with this dynamic optimisation problem many times now, so we will skip the steps and simply write down the result.
The familiar consumption path is given by

$$\frac{\dot{C}_t}{C_t} = R_t - \delta - \rho$$

which determines the path of consumption chosen by the household.

Final Good I

- There is a representative firm that produces the final good Y_t.
- This firm hires labour L_t and uses intermediate goods as inputs.

The production function is different from before and given by

$$Y_t = L_t^{1-\alpha} \sum_{i=1}^{N} X_t^{\alpha}(i) = L_t^{1-\alpha} \left[X_t^{\alpha}(1) + X_t^{\alpha}(2) + \cdots + X_t^{\alpha}(N) \right]$$

- the parameter $\alpha \in (0, 1)$ determines the degree of labour intensity $1 - \alpha$ in production.
- In other words, there are N different intermediate goods $X_t(i)$ that are indexed by $i \in [1, N]$.

Final Good II

The profit function for this final-good firm:

$$\Pi_t = Y_t - W_t L_t - \sum_{i=1}^{N} P_t(i) X_t(i)$$

where we have implicitly chosen Y_t as the numeraire and $P_t(i)$ is the price of $X_t(i)$ for $i \in [1, N]$.

Differentiating the profit function with respect to L_t and $X_t(i)$ yields

$$\frac{\partial \Pi_t}{\partial L_t} = (1-\alpha) L_t^{-\alpha} \sum_{i=1}^{N} X_t^{\alpha}(i) - W_t = (1-\alpha)\frac{Y_t}{L_t} - W_t$$
$$\frac{\partial \Pi_t}{\partial X_t(i)} = \alpha L_t^{1-\alpha} X_t^{\alpha-1}(i) - P_t(i) = 0$$

These two sets of equations are the demand functions for L_t and $X_t(i)$ for $i \in [1, N]$.

Intermediate Goods I

- Each variety of intermediate goods is produced by a firm that has monopolistic power over this variety.
- We refer to each variety of intermediate goods as an industry, and let's consider an arbitrary industry i.
- In industry i, the monopolistic firm rents capital from the household to produce intermediate good i.

We consider a simple production function given by

$$X_t(i) = K_t(i)$$

In other words, one unit of capital produces one unit of intermediate good i.

Intermediate Goods II

The profit function for intermediate-good firm i is

$$\pi_t(i) = P_t(i)X_t(i) - R_tK_t(i)$$

Given that this firm acts as a monopolist, it chooses its price to maximise profit (rather than taking the price as given). Substituting FOC-2 and production function into profit function for intermediate-good firm yields

$$\pi_t(i) = \alpha L_t^{1-\alpha}X_t^{\alpha}(i) - R_tX_t(i).$$

Differentiating $\pi_t(i)$ with respect to $X_t(i)$ yields

$$\frac{\partial \pi_t(i)}{\partial X_t(i)} = \alpha \underbrace{\alpha L_t^{1-\alpha}X_t^{\alpha-1}(i)}_{=P_t(i)} - R_t = 0.$$

Intermediate Goods III

- Using FOC-2, we can re-express last equation as $P_t(i) = R_t/\alpha > R_t$, where $1/\alpha > 1$ is the markup ratio.
- Due to this markup pricing, the intermediate-good sector generates positive monopolistic profits.
- As a result of these monopolistic profits, the decentralised economy has a different allocation of resources compared with the centralised economy as we show below.

Aggregation I

$$\frac{\partial \pi_t(i)}{\partial X_t(i)} = \alpha \underbrace{\alpha L_t^{1-\alpha} X_t^{\alpha-1}(i)}_{=P_t(i)} - R_t = 0.$$

This equation shows that $X_t(i) = \left(\alpha^2/R_t\right)^{1/(1-\alpha)} L_t$ is the same across all $i \in [1, N]$.

Using this information, we impose symmetry on final-good production function to obtain

$$Y_t = L_t^{1-\alpha} N X_t^{\alpha}(i)$$

Then, we substitute the resource constraint on capital given by $X_t(i) = K_t(i) = K_t/N$ into equation above to derive the aggregate production function:

$$Y_t = L_t^{1-\alpha} N \left(\frac{K_t}{N}\right)^{\alpha} = N^{1-\alpha} K_t^{\alpha} L_t^{1-\alpha} = A K_t^{\alpha} L_t^{1-\alpha}$$

Aggregation II

$$Y_t = L_t^{1-\alpha} N \left(\frac{K_t}{N} \right)^{\alpha} = N^{1-\alpha} K_t^{\alpha} L_t^{1-\alpha} = A K_t^{\alpha} L_t^{1-\alpha}$$

where we have relabelled $N^{1-\alpha}$ as A.

- This aggregate production function is the same as before.
- In this economy, it is the number of differentiated products that determines the level of technology; in other words, a higher level of technology is driven by a larger number of differentiated products.

Aggregation III

Setting $L_t = 1$ in FOC-2 yields

$$P_t(i) = \alpha X_t^{\alpha-1}(i)$$

We know that $P_t(i) = R_t/\alpha$. Substituting this equation and the resource constraint $X_t(i) = K_t/N$ into $P_t(i) = \alpha X_t^{\alpha-1}(i)$ yields the market demand curve for capital as follows:

$$R_t = \alpha^2 \left(\frac{K_t}{N}\right)^{\alpha-1} = \alpha^2 A K_t^{\alpha-1}$$

where the second equality follows from $N^{1-\alpha} = A$.

Aggregation IV

We impose steady state on the household's consumption path to derive the long-run capital supply curve as

$$\frac{\dot{C}_t}{C_t} = 0 \Leftrightarrow R_t = \rho + \delta$$

(Figure 12.1).

Finally, combining capital demand and capital supply yields the steady-state equilibrium level of capital under monopolistic competition as

$$K_m^* = \left(\frac{\alpha^2 A}{\rho + \delta} \right)^{1/(1-\alpha)}$$

Aggregation V

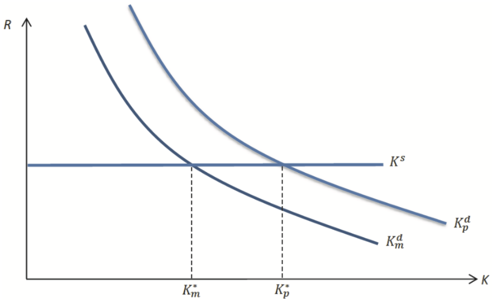

Figure 12.1. Monopolistic competition vs. perfect competition.

Monopolistic Competition vs. Perfect Competition I

It may seem that the Ramsey model with a monopolistically competitive market is very different from the Ramsey model with a perfectly competitive market.

However, the two models have the same aggregate structure:
- the same aggregate production function
- the same capital-accumulation equation
- the same consumption path (except that the equilibrium rental price of capital differs across the two models).

We will use MPK_t to denote the marginal product of capital.

Monopolistic Competition vs. Perfect Competition II

Regardless of the underlying market structure, the consumption path in the Ramsey model is

$$\frac{\dot{C}_t}{C_t} = R_t - \delta - \rho$$

In a perfectly competitive market, the capital demand curve is $R_t = MPK_t = \alpha A K_t^{\alpha-1}$, which yields the following steady-state equilibrium level of capital:

$$K_p^* = \left(\frac{\alpha A}{\rho + \delta}\right)^{1/(1-\alpha)}$$

see Figure 12.1.

Monopolistic Competition vs. Perfect Competition III

In a monopolistically competitive market, the capital demand curve is $R_t = \alpha MPK_t = \alpha^2 AK_t^{\alpha-1}$, which yields the following steady-state equilibrium level of capital:

$$K_m^* = \left(\frac{\alpha^2 A}{\rho + \delta}\right)^{1/(1-\alpha)} < K_p^*$$

- In other words, the steady-state equilibrium level of capital differs in the two economies.
- The reason is that under monopolistic competition, the rental price of capital is less than the marginal product of capital (i.e., $R_t = \alpha MPK_t < MPK_t$ given $\alpha < 1$).
- Intuitively, due to monopolistic distortion, the market return to capital is lower under monopolistic competition than under perfect competition; as a result, the household accumulates less capital.

Monopolistic Competition vs. Perfect Competition IV

Finally, we derive the steady-state saving rate given by

$$s^* \equiv \frac{I^*}{Y^*} = \frac{\delta K^*}{A\,(K^*)^{\alpha}} = \frac{\delta\,(K^*)^{1-\alpha}}{A}$$

In the Ramsey model with a monopolistically competitive market, the household's saving rate s_m^* is given by

$$s_m^* = \frac{\alpha^2 \delta}{\rho + \delta} < \frac{\alpha\delta}{\rho + \delta} = s_p^*.$$

Given an empirically relevant range of capital intensity $\alpha \in [1/3, 1/2]$, the distortion of monopolistic competition can cause the household's saving rate to be less than half of the optimal level.

Monopolistic Competition vs. Perfect Competition V

This distortion on the saving rate in turn reduces the level of output significantly; to see this,

$$\frac{Y_m^*}{Y_p^*} = \frac{A\left(K_m^*\right)^{\alpha}}{A\left(K_p^*\right)^{\alpha}} = \alpha^{\alpha/(1-\alpha)}$$

For example, if $\alpha = 1/2$, then the monopolistic level of output Y_m^* is only half of the competitive level of output Y_p^*.

Market Power of Monopolistic Firms I

Suppose the government imposes an upper bound on the monopolistic price given by

$$P_t(i) = \mu R_t < R_t/\alpha,$$

where the policy parameter $\mu \in (1, 1/\alpha)$ determines the markup ratio, capturing the market power of monopolistic firms.

Substituting $P_t(i) = \mu R_t < R_t/\alpha$ and the resource constraint $X_t(i) = K_t/N$ into $P_t(i) = \alpha X_t^{\alpha-1}(i)$ yields

$$R_t = \frac{\alpha}{\mu}\left(\frac{K_t}{N}\right)^{\alpha-1} = \frac{\alpha A K_t^{\alpha-1}}{\mu} = \frac{MPK_t}{\mu}$$

where the second equality follows from $N^{1-\alpha} = A$.

Market Power of Monopolistic Firms II

$$R_t = \frac{\alpha}{\mu}\left(\frac{K_t}{N}\right)^{\alpha-1} = \frac{\alpha A K_t^{\alpha-1}}{\mu} = \frac{MPK_t}{\mu}$$

This final equation shows that the difference between the marginal product of capital and its rental price is increasing in μ, which in turn determines the degree of distortion in the market economy.

Substituting the consumption path into last equation yields the steady-state equilibrium level of capital given by

$$K_m^* = \left[\frac{\alpha A}{\mu(\rho+\delta)}\right]^{1/(1-\alpha)} < \left(\frac{\alpha A}{\rho+\delta}\right)^{1/(1-\alpha)}$$

unless $\mu \to 1$.

Market Power of Monopolistic Firms III

Similarly, the household's saving rate is given by

$$s_m^* \equiv \frac{I^*}{Y^*} = \frac{\delta K^*}{A\,(K^*)^{\alpha}} = \frac{\delta\,(K^*)^{1-\alpha}}{A} = \frac{\alpha\delta}{\mu(\rho+\delta)} < \frac{\alpha\delta}{\rho+\delta},$$

unless $\mu \to 1$.

In other words, as μ approaches unity, K_m^* and s_m^* under monopolistic competition coincide with their equilibrium levels under perfect competition.

Therefore, the market equilibrium allocations in the decentralised Ramsey model with monopolistic competition can be socially optimal if the government completely removes the market power of monopolistic firms by restricting the markup ratio μ to unity.

Summary I

- We introduce an alternative market structure into the Ramsey model.
- Specifically, we consider a monopolistically competitive product market.
- In this case, each of the monopolistic firms sells a differentiated product.
- As a result, the monopolistic firms have market power and charge a markup over their marginal cost of production.
- This monopolistic distortion causes the rental price to be less than the marginal product of capital.
- As a result, the household accumulates less capital and chooses a lower saving rate than the socially optimal level.
- In other words, due to the presence of monopolistic distortion in the economy, the market equilibrium is inefficient.

Summary II

- Therefore, the first fundamental theorem of welfare economics does not hold in the Ramsey model with a monopolistically competitive product market unless the government removes the monopolistic distortion in the economy.

The Romer Model I

- In the model, endogenous technological change by developing a growth model in which technological progress is driven by the invention of new products,
- This invention is due to research and development (R&D) by profit-seeking entrepreneurs.
- In the previous chapter, we introduced a monopolistically competitive market structure to the Ramsey model.
- If we further introduce an R&D sector into the model to allow for endogenous growth in the number of products, then we have the Romer model.
- Once we derive the endogenous growth rate of technology in the Romer model, we can then perform comparative statics to explore the determinants of technological progress.

The Romer Model II

- There is a representative firm that produces the final good.
- There are a number of firms that produce differentiated intermediate goods.
- The novel element here is that the number of these differentiated goods increases over time due to innovation.
- Because each of these intermediate-good firms sells a differentiated product, it generates monopolistic profits, which serve as the incentives for R&D.

The Romer Model III

The model has the following components:

- a representative household
- a representative firm that produces the final good
- a number of monopolistic firms that produce differentiated intermediate goods
- competitive R&D entrepreneurs who invest in R&D and create new varieties of intermediate goods.

Household I

The representative household has the following lifetime utility function:

$$U = \int_0^{\infty} e^{-\rho t} \ln C_t dt$$

- The parameter $\rho > 0$ is the household's discount rate and C_t is the level of consumption at time t.
- The household inelastically supplies L units of labour to earn a wage income W_t.
  (we consider L units of labour supply instead of 1 unit)
- Furthermore, it accumulates capital K_t and rents it to firms to earn a capital-rental income R_t.
- Here, we also introduce financial asset F_t (i.e., the shares of monopolistic firms) and its rate of return given by r_t, which is also the real interest rate.

Household II

Then, the asset-accumulation equation becomes

$$\dot{F}_t + \dot{K}_t = r_t F_t + R_t K_t + W_t L - C_t - \delta K_t,$$

where the parameter $\delta > 0$ is the depreciation rate of capital.

Note: the asset-accumulation equation can be re-expressed as $\dot{K}_t = Y_t - C_t - \delta K_t$ in equilibrium.

Household III

The Hamiltonian function:

$$H_t = \ln C_t + \lambda_t \left(r_t F_t + R_t K_t + W_t L - C_t - \delta K_t \right).$$

The FOCs

$$\frac{\partial H_t}{\partial C_t} = \frac{1}{C_t} - \lambda_t = 0,$$
$$\frac{\partial H_t}{\partial K_t} = \lambda_t \left(R_t - \delta \right) = \lambda_t \rho - \dot{\lambda}_t,$$
$$\frac{\partial H_t}{\partial F_t} = \lambda_t r_t = \lambda_t \rho - \dot{\lambda}_t,$$

where F_t is also a state variable.

Household IV

Combining FOC-2 and FOC-3 yields a no-arbitrage condition given by $r_t = R_t - \delta$.

In other words, the two types of assets (F_t and K_t) must yield the same rate of return.

We have the consumption path given by

$$\frac{\dot{C}_t}{C_t} = r_t - \rho$$

where $r_t = R_t - \delta$ from the no-arbitrage condition.

Usually, the consumption path is expressed in r_t instead of $R_t - \delta$.

Final Good I

There is a representative firm that produces the final good Y_t.
This firm hires labour and uses intermediate goods $X_t(i)$ as inputs.
The production function :

$$Y_t = L_{Y,t}^{1-\alpha} \int_0^{N_t} X_t^{\alpha}(i) di,$$

- the parameter $\alpha \in (0,1)$ determines the intensity $1-\alpha$ of production labour.
- $L_{Y,t}$ is the number of production workers
(and we will use $L_{R,t}$ to denote R&D workers such that the resource constraint on labour is given by $L_{Y,t} + L_{R,t} = L$).
- It is useful to note that we are treating the number of varieties N_t as a continuous number (instead of a discrete number) for modelling simplicity.

Final Good II

The profit function for the final-good firm:

$$\Pi_t = Y_t - W_t L_{Y,t} - \int_0^{N_t} P_t(i) X_t(i) di$$

where we have implicitly chosen Y_t as the numeraire and $P_t(i)$ is the price of $X_t(i)$ for $i \in [1, N]$.

Final Good III

Differentiating Π_t with respect to $L_{Y,t}$ and $X_t(i)$ yields

$$\begin{aligned}\frac{\partial \Pi_t}{\partial L_{Y,t}} &= (1-\alpha)L_{Y,t}^{-\alpha}\int_0^{N_t} X_t^{\alpha}(i)di - W_t \\ &= (1-\alpha)\frac{Y_t}{L_{Y,t}} - W_t = 0, \\ \frac{\partial \Pi_t}{\partial X_t(i)} &= \alpha L_{Y,t|}^{1-\alpha} X_t^{\alpha-1}(i) - P_t(i) = 0,\end{aligned}$$

for $i \in [0, N_t]$.

These two sets of equations are the demand functions for $L_{Y,t}$ and $X_t(i)$ for $i \in [0, N_t]$.

Intermediate Goods I

- Each variety of intermediate goods is produced by a firm that has monopolistic power over this variety.
- We will refer to each variety of intermediate goods as an industry, and let's consider an arbitrary industry i.
- In industry i, the monopolistic firm rents capital from the household to produce the intermediate good.

We consider a simple production function as before.

$$X_t(i) = K_t(i)$$

In other words, one unit of capital produces one unit of intermediate good.

Intermediate Goods II

The profit function for this intermediate-good firm:

$$\pi_t(i) = P_t(i)X_t(i) - R_t K_t(i).$$

Given that this firm acts as a monopolist, it chooses its price to maximise profit (rather than taking the price as given).

Substituting $\frac{\partial \Pi_t}{\partial X_t(i)}$ and $X_t(i)$ into $\pi_t(i)$ yields

$$\pi_t(i) = \alpha L_{Y,t}^{1-\alpha} X_t^{\alpha}(i) - R_t X_t(i)$$

Differentiating $\pi_t(i)$ with respect to $X_t(i)$ yields

$$\frac{\partial \pi_t(i)}{\partial X_t(i)} = \alpha \underbrace{\alpha L_{Y,t}^{1-\alpha} X_t^{\alpha-1}(i)}_{=P_t(i)} - R_t = 0$$

Intermediate Goods III

Using $\frac{\partial \Pi_t}{\partial X_t(i)}$, we can re-express $\frac{\partial \pi_t(i)}{\partial X_t(i)}$ as $P_t(i) = R_t/\alpha > R_t$, where $1/\alpha > 1$ is the markup ratio.

Due to this markup pricing, the intermediate-good sector generates positive monopolistic profits.

R &D I

The novel element in the Romer model is the R&D sector.
The law of motion for the number of varieties is given by

$$\dot{N}_t = \theta N_t L_{R,t},$$

where $L_{R,t}$ is the number of R&D workers and $\theta > 0$ is an R&D productivity parameter.

The growth rate of N_t given by $g_N \equiv \dot{N}_t / N_t = \theta L_{R,t}$;
therefore, increasing R&D labour stimulates economic growth.

However, what determines the equilibrium allocation of R&D labour $L_{R,t}$?

R &D II

Let v_t denote the market value of a new invention (i.e., a new variety of differentiated products).
The market value of creating $\dot{N}_t$ new inventions is $\dot{N}_t v_t$.
The cost of $\mathrm{R\&D}$ is $W_t L_{R,t}$.

Given that there is free entry in the $\mathrm{R\&D}$ sector, this sector generates zero profit such that

$$\dot{N}_t v_t = W_t L_{R,t}$$

Substituting $\dot{N}_t$ into $\dot{N}_t v_t$, we obtain

$$\theta N_t v_t = W_t,$$

where we have cancelled $L_{R,t}$ from both sides.

The last equation determines the equilibrium allocation of $R\&D$ labour.

R &D III

To show this result, we will need to find out the value of v_t.
The value of an invention is the present value of all the future monopolistic profits. Formally,

$$v_t = \frac{\pi_t}{r - g_\pi},$$

where g_π is the steady-state equilibrium growth rate of π_t.
Substituting v_t yields

$$\frac{\theta N_t \pi_t}{r - g_\pi} = W_t$$

We already know that $W_t = (1-\alpha)Y_t/L_{Y,t}$.
We also know that $r = \rho + g_C$, where g_C is the steady-state equilibrium growth rate of C_t.
Therefore, what we need to do next is to derive an expression for π_t.

Aggregation I

Equation

$$\frac{\partial \pi_t(i)}{\partial X_t(i)} = \alpha \underbrace{\alpha L_{Y,t}^{1-\alpha} X_t^{\alpha-1}(i)}_{=P_t(i)} - R_t = 0$$

implies that $X_t(i) = \left(\alpha^2/R_t\right)^{1/(1-\alpha)} L_{Y,t}$ is the same across all $i \in [0, N_t]$.

Using this information, we impose symmetry on

$$Y_t = L_{Y,t}^{1-\alpha} \int_0^{N_t} X_t^{\alpha}(i) di,$$

to obtain $Y_t = L_{Y,t}^{1-\alpha} N_t X_t^{\alpha}(i)$.

Aggregation II

Then, we substitute into this equation the resource constraint on capital given by $X_t(i) = K_t(i) = K_t/N_t$ to derive the aggregate production function:

$$Y_t = L_{Y,t}^{1-\alpha} N_t \left(\frac{K_t}{N_t}\right)^{\alpha} = N_t^{1-\alpha} K_t^{\alpha} L_{Y,t}^{1-\alpha} = A_t K_t^{\alpha} L_{Y,t}^{1-\alpha}$$

where we have relabelled $N_t^{1-\alpha}$ as A_t.

Therefore, the growth rate of A_t in the Solow model becomes $g_A = (1-\alpha)g_N$, which is endogenous in the Romer model.
Also, it should be noted that the aggregate production function is the same as the one in the Solow model except that L_t is replaced by $L_{Y,t}$.

Aggregation III

Recall that the profit function for the intermediate-good firm i is

$$\pi_t(i) = P_t(i)X_t(i) - R_t K_t(i).$$

Substituting $X_t(i) = K_t(i)$ and $P_t(i) = R_t/\alpha$ into the profit function yields

$$\pi_t = \frac{1-\alpha}{\alpha} R_t K_t(i) = \frac{1-\alpha}{\alpha} \frac{R_t K_t}{N_t}$$

which uses the resource constraint $X_t(i) = K_t(i) = K_t/N_t$.

Aggregation IV

Recall that the production labour income share of output is $W_t L_{Y,t} = (1-\alpha) Y_t$, which implies that the remaining share of output αY_t goes to capital income and monopolistic profit:

$$\pi_t N_t + R_t K_t = N_t P_t X_t = Y_t.$$

Note that GDP in this economy is given by $GDP_t = Y_t + R\&D_t = Y_t + W_t L_{R,t}$.

Combining last two profit functions yields the profit share of output given by

$$\pi_t = \alpha(1-\alpha)\frac{Y_t}{N_t},$$

which implies that $g_\pi = g_Y - g_N$.

Solving the Model I

Using $\pi_t = \alpha(1-\alpha)\frac{Y_t}{N_t}$ along with $W_t = (1-\alpha)Y_t/L_{Y,t}$ and $r = \rho + g_C$, we can simplify $\frac{\theta N_t \pi_t}{r - g_\pi} = W_t$ to

$$\frac{\alpha\theta}{\rho + g_C - g_Y + g_N} = \frac{1}{L_Y}.$$

On the balanced growth path, the growth rates of output and consumption are the same such that $g_C = g_Y$.

Using $g_N = \theta L_R$, we can further simplify last equation to

$$\frac{\alpha\theta}{\rho + \theta L_R} = \frac{1}{L_Y}.$$

Solving the Model II

Combining this equation with the resource constraint on labour $L_Y + L_R = L$ yields

$$L_R = \frac{\alpha}{1+\alpha}\left(L - \frac{\rho}{\alpha\theta}\right).$$

Therefore, the equilibrium allocation of R&D labour has the following comparative statics:

$$L_R(\rho, \underset{-}{\theta}, \underset{+}{\alpha}, \underline{+}).$$

The intuition of these comparative statics results can be explained as follows.

R&D productivity I

- An improvement in R&D productivity θ increases the growth rate g_N of technology for a given level of R&D labour L_R and also makes R&D labour more productive, which in turn increases R&D labour in the economy.
- Therefore, g_N is increasing in θ. To see this, recall that the growth rate of technology is $g_N = \theta L_R(\theta)$.
- Therefore, an increase in R&D productivity θ has a direct positive effect on g_N by increasing R&D productivity and also an indirect positive effect by increasing R&D labour L_R.
- The parameter θ captures the importance of human capital on the innovation capacity of an economy.
- To stimulate economic growth, policymakers could consider devoting more resources to education that improves the innovative capacity of entrepreneurs, scientists and engineers.

Discount rate I

- A higher discount rate ρ increases the real interest rate and decreases the present value of monopolistic profits as well as the value of inventions, which in turn decreases R&D labour in the economy.
- Therefore, g_N is decreasing in ρ. To see this, recall that the growth rate of technology is $g_N = \theta L_R(\rho)$.
- Therefore, an increase in the discount rate ρ decreases the growth rate of technology by decreasing $\mathrm{R\&D}$ labour L_R.
- The parameter ρ captures the effects of household preference and also frictions in the financial market, such as credit constraints, on innovation.
- To stimulate economic growth, policymakers could consider policies that improve the efficiency of the financial market.

Capital and labour intensity in production I

- An increase in α increases capital intensity and reduces labour intensity in the production process, allowing more labour to be devoted to R&D.
- Therefore, g_N is increasing in α. To see this, recall that the growth rate of the number of differentiated products is $g_N = \theta L_R(\alpha)$.
- Therefore, an increase in capital intensity α increases the growth rate of technology by increasing R&D labour L_R.
- The parameter α captures the effect of structural transformation of an economy from a labour-intensive production process to a capital-intensive production process.
- However, the positive effect of α on R&D is based on the assumption that R&D does not require the use of capital.

Capital and labour intensity in production II

- If R&D also uses capital, then the effect of α on growth may be reversed depending on the degree of capital intensity in the R&D process.

Labour Force I

- A larger labour force L increases the supply of labour in the economy, which in turn increases $\mathrm{R\&D}$ labour and the growth rate of technology.
- Therefore, g_N is increasing in L. To see this, recall that the growth rate of technology is $g_N = \theta L_R(L)$.
- Therefore, an increase in the labour force L increases the growth rate of technology by increasing $\mathrm{R\&D}$ labour L_R.
- In the literature, this is known as the scale effect, which is often viewed as a counterfactual implication of the Romer model.

Summary I

- We develop the Romer model by introducing an R&D sector into the Ramsey model with a monopolistically competitive product market.
- In this case, the positive monopolistic profit in the economy serves as an incentive for entrepreneurs to do R&D, which in turn gives rise to technological progress.
- We derive the market equilibrium level of R&D labour, which determines the equilibrium growth rate of technology.
- In summary, the market equilibrium level of R&D labour is increasing in the monopolistic power of firms, the productivity of R&D, the degree of capital intensity in production and the size of the labour force but decreasing in the discount rate of the representative household.
- Therefore, cross-country variation in these determinants helps to explain some of the variation in economic growth across countries.