Policy Analysis in Neoclassical Growth Model

Bilgin Bari

Macroeconomic Theory

1. Fiscal Policy
   - Government Spending
   - Labour Income Tax
   - Capital Income Tax

2. Monetary Policy
   - New Keynesian Model

Government Spending I

- The NGM also allows us to perform policy analysis.
- We can analyse government policies in the NGM.
- We consider a number of fiscal policy instruments.
- The first policy instrument is government spending (permanent).
- Specifically, we analyse the macroeconomic effects of changes in government spending in the NGM with elastic labour supply.
- The expansionary effects of government spending operate through an income effect on labour supply.

Household I

The representative household's utility function:

$$U = \int_0^\infty e^{-\rho t} \left[\ln C_t + \beta \ln (L - l_t)\right] dt$$

- $\rho > 0$ is the household's discount rate
- $\beta > 0$ determines the importance of leisure $L - l_t$ relative to consumption C_t in the utility function.
- L is the household's total labour endowment
- l_t is the level of employment chosen by the household.
- The household elastically supplies l_t units of labour to earn a wage income W_t.
- It accumulates capital K_t and rents it to the representative firm to earn a capital-rental income R_t.

Household II

The asset-accumulation equation:

$$\dot{K}_t = R_t K_t + W_t l_t - C_t - T_t$$

- The capital depreciation rate is zero.

- T_t is a lump-sum tax.

Government

- The government collects tax revenue T_t to pay for government spending G_t.
- The balanced budget condition is $G_t = T_t$.
- We define the ratio of government spending to output as $\gamma \equiv G_t / Y_t$.
- We are interested in the effects of changes in γ on other macroeconomic variables.

Hamiltonian I

The Hamiltonian function of the household:

$$H_t = \ln C_t + \beta \ln (L - l_t) + \lambda_t (R_t K_t + W_t l_t - C_t - T_t).$$

The first-order conditions:

$$\frac{\partial H_t}{\partial l_t} = -\frac{\beta}{L - l_t} + \lambda_t W_t = 0$$
$$\frac{\partial H_t}{\partial C_t} = \frac{1}{C_t} - \lambda_t = 0$$
$$\frac{\partial H_t}{\partial K_t} = \lambda_t R_t = \lambda_t \rho - \dot{\lambda}_t$$

Recall that K_t is a state variable (i.e., a variable that accumulates over time), so we have to set $\partial H_t / \partial K_t = \lambda_t \rho - \dot{\lambda}_t$.

Hamiltonian II

Combining $\frac{\partial H_t}{\partial l_t}$ and $\frac{\partial H_t}{\partial C_t}$ yields the labour supply curve l_t^s :

$$l_t^s = L - \frac{\beta C_t}{W_t},$$

- increasing in the wage rate W_t (i.e., a substitution effect)
- decreasing in consumption C_t (i.e., an income effect).

Unless $\beta = 0$, unemployment $L - l_t^s$ is positive.

Taking the log of $\frac{\partial H_t}{\partial C_t}$ and substituting it into $\frac{\partial H_t}{\partial K_t}$ yields the optimal consumption path:

$$\frac{\dot{C}_t}{C_t} = R_t - \rho.$$

Note that the labour supply curve and the optimal consumption path are the same as before.

Firm I

There is a representative firm in the economy, and this firm hires labour l_t and rents capital K_t from the household to produce output Y_t using the following Cobb-Douglas production function:

$$Y_t = AK_t^{\alpha} l_t^{1-\alpha}$$

- $\alpha \in (0, 1)$ is the degree of capital intensity in production
- A is the exogenous level of technology.

The profit function Π_t :

$$\Pi_t = Y_t - R_t K_t - W_t l_t,$$

* We have implicitly chosen Y_t as the numeraire (i.e., the price of Y_t is normalised to unity).

Firm II

Differentiating profit function with respect to K_t and l_t :

$$\frac{\partial \Pi_t}{\partial K_t} = \frac{\partial Y_t}{\partial K_t} - R_t = \alpha A \left(\frac{l_t}{K_t} \right)^{1-\alpha} - R_t = 0$$
$$\frac{\partial \Pi_t}{\partial l_t} = \frac{\partial Y_t}{\partial l_t} - W_t = (1-\alpha) A \left(\frac{K_t}{l_t} \right)^{\alpha} - W_t = 0$$

These two equations are the demand functions for K_t and l_t.

Note that the demand functions for K_t and l_t are also the same as before.

Long-Run Effects of Government Spending I

We start with the long-run effects of permanent changes in government spending.

In the long run, the level of capital fully adjusts to its steady-state equilibrium level.

Recall that the optimal consumption path is given by

$$\frac{\dot{C}_t}{C_t} = R_t - \rho$$

In the steady state, we have $\dot{C}_t = 0$.

Therefore, the long-run supply curve of capital is perfectly elastic and given by

$$R_t = \rho$$

Long-Run Effects of Government Spending II

The labour supply curve:

$$W_t = \frac{\beta C_t}{L - l_t}$$

The demand curves of capital and labour:

$$R_t = \alpha A \left(\frac{l_t}{K_t}\right)^{1-\alpha} = \alpha \frac{Y_t}{K_t}$$

$$W_t = (1-\alpha) A \left(\frac{K_t}{l_t}\right)^{\alpha} = (1-\alpha) \frac{Y_t}{l_t}$$

Long-Run Effects of Government Spending III

Combining labour supply and labour demand:

$$l_t = L - \frac{\beta C_t}{W_t} = L - \frac{\beta C_t}{(1-\alpha)Y_t} l_t$$

Given the assumption of a zero capital depreciation rate (i.e., $\delta = 0$), the steady-state equilibrium level of investment I^* is zero.

Therefore, the steady-state equilibrium level of consumption is given by

$$C^* = Y^* - G^* = (1-\gamma)Y^*,$$

which is proportional to the steady-state equilibrium level of output.

Long-Run Effects of Government Spending IV

Substituting C^* into l_t yields the steady-state equilibrium level of labour l^* :

$$l^* = \frac{L}{1 + \beta(1-\gamma)/(1-\alpha)}$$

which is increasing in the government-spending ratio γ.

- Intuitively, an increase in government spending γ raises tax T and reduces the after-tax income to cause a negative income effect on the household, which then consumes less leisure and supplies more labour l.
- Graphically, it shifts the labour supply curve to the right; as a result, the equilibrium level of labour l increases and the wage rate W decreases (Figure 5.1).
- In the capital market, the increase in the level of labour shifts the capital demand curve to the right.

Long-Run Effects of Government Spending V

- Given the horizontal long-run capital supply curve, the rental price R remains at the initial level whereas the equilibrium level of capital increases in the long run (Figure 5.2).
- The increase in capital shifts the labour demand curve to the right. As a result, the wage rate W increases and returns to the initial level because the capital-labour ratio K/l is independent of γ .
- Finally, the production function $Y_t = AK_t^{\alpha} l_t^{1-\alpha}$ implies that the increases in labour and capital both give rise to an increase in the steady-state equilibrium level of output Y^*.

Long-Run Effects of Government Spending VI

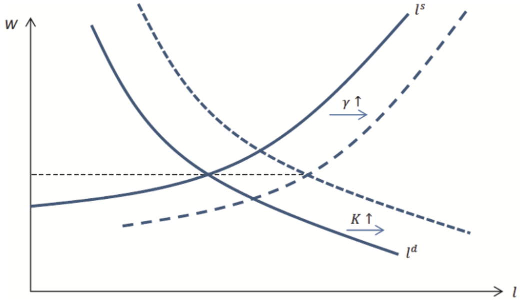

Figure 5.1. Labour market in the long run.

Long-Run Effects of Government Spending VII

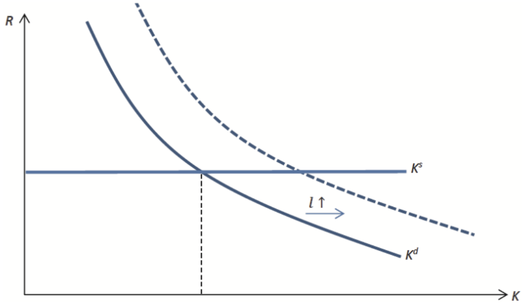

Figure 5.2. Capital market in the long run.

Long-Run Effects of Government Spending VIII

The long-run effects of government spending γ can be summarised as follows:

Long-run effects of an increase in γ				
Y	K	R	l	W
increase	increase	no change	increase	no change

Short-Run Effects of Government Spending I

- To complete our analysis, we now look at the short-run effects of permanent changes in government spending.
- We define the short run as the moment when the parameter γ changes.
- At this moment, the level of capital in the economy is predetermined. In other words, the short-run supply curve of capital is vertical.
- As before, an increase in government spending γ raises tax and reduces the after-tax income to cause a negative income effect on the household, which then consumes less leisure and supplies more labour.
- Graphically, it shifts the labour supply curve to the right; as a result, the equilibrium level of labour increases and the wage rate decreases (Figure 5.3).

Short-Run Effects of Government Spending II

- In the capital market, the increase in the level of labour shifts the capital demand curve to the right.
- Given the vertical short-run capital supply curve, the rental price increases whereas the equilibrium level of capital remains unchanged in the short run (Figure 5.4).
- Finally, the production function $Y_t = AK_t^{\alpha} l_t^{1-\alpha}$ implies that the increase in labour gives rise to an increase in the level of output.

Short-Run Effects of Government Spending III

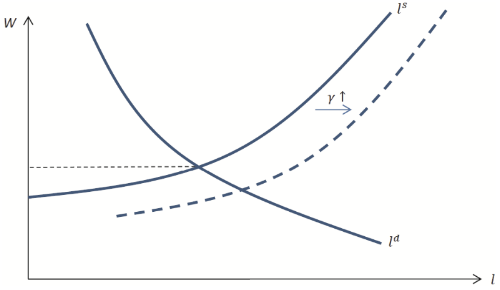

Figure 5.3. Labour market in the short run.

Short-Run Effects of Government Spending IV

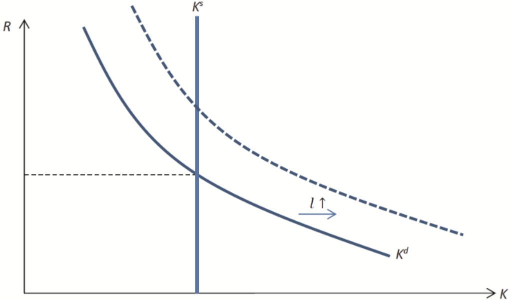

Figure 5.4. Capital market in the short run.

Short-Run Effects of Government Spending V

The short-run effects of government spending γ :

Short-run effects of an increase in γ				
Y	K	R	l	W
increase	no change	increase	increase	decrease

Summary I

- We explore the effects of permanent changes in the level of government spending in the NGM with elastic labour supply.
- We find that an increase in the level of government spending is accompanied by an increase in taxation, which in turn gives rise to an income effect on labour supply and shifts the labour supply curve to the right.
- In the short run, the equilibrium level of labour increases and the wage rate decreases while the level of output increases.
- The increase in labour causes a general-equilibrium effect on the capital market by shifting the capital demand curve to the right.
- As a result, the rental price of capital increases because the short-run capital supply curve is perfectly inelastic.

Summary II

- Given that the capital supply curve becomes perfectly elastic in the long run, the equilibrium level of capital increases and in turn affects the labour market by shifting the labour demand curve to the right.
- At the end, the equilibrium level of labour increases by an even larger amount whereas the wage rate returns to the initial level.
- In summary, an increase in the level of government spending has an expansionary effect on the macroeconomy and increases the levels of output, capital and labour without affecting the rental price and the wage rate in the long run.

Labour Income Tax I

- We considered a lump-sum tax, which is a very rare fiscal policy instrument in reality.
- Therefore, we now consider a labour income tax, which is a more realistic tax instrument.
- Once again, we use in this policy analysis the NGM with elastic labour supply.
- The contractionary effects of labour income tax operate through a substitution effect on labour supply.

Household I

The household's utility function:

$$U = \int_0^\infty e^{-\rho t} \left[\ln C_t + \beta \ln \left(L - l_t \right) \right] dt,$$

- $\rho > 0$ is the household's discount rate
- $\beta > 0$ determines the importance of leisure $L - l_t$ relative to consumption C_t in the utility function.
- L is the household's total labour endowment, and l_t is the level of employment chosen by the household.
- The household elastically supplies l_t units of labour to earn an after-tax wage income $(1 - \tau_W)\, W_t$, where $\tau_W > 0$ is the tax rate on labour income.
- The household accumulates capital K_t and rents it to the representative firm to earn a capital-rental income R_t.

Household II

The asset-accumulation equation is

$$\dot{K}_t = R_t K_t + (1 - \tau_W)\, W_t l_t - C_t - T_t,$$

- The capital depreciation rate is zero
- T_t is a lump-sum tax.

Government I

- The government collects tax revenue to pay for government spending G_t.
- The balanced budget condition is $G_t = T_t + \tau_W W_t l_t$.
- We define the ratio of government spending to output as $\gamma \equiv G_t / Y_t$.
- We are interested in the effects of changes in the labour income tax rate τ_W on other macroeconomic variables.
- We previously saw that changes in G_t cause an income effect on the household.
- To separate this income effect from our analysis, we assume that changes in the labour income tax rate τ_W are balanced by changes in the lump-sum tax T_t while the government-spending ratio γ does not change.
- Therefore, changes in the labour income tax rate τ_W only give rise to a substitution effect on the household's labour supply.

Hamiltonian I

The Hamiltonian function of the household:

$$H_t = \ln C_t + \beta \ln (L - l_t) + \lambda_t [R_t K_t + (1 - \tau_W) W_t l_t - C_t - T_t].$$

The first-order conditions:

$$\frac{\partial H_t}{\partial l_t} = -\frac{\beta}{L - l_t} + \lambda_t (1 - \tau_W) W_t = 0,$$
$$\frac{\partial H_t}{\partial C_t} = \frac{1}{C_t} - \lambda_t = 0,$$
$$\frac{\partial H_t}{\partial K_t} = \lambda_t R_t = \lambda_t \rho - \dot{\lambda}_t.$$

Recall that K_t is a state variable (i.e., a variable that accumulates over time), so we have to set $\partial H_t / \partial K_t = \lambda_t \rho - \dot{\lambda}_t$.

Hamiltonian II

Combining $\frac{\partial H_t}{\partial l_t}$ and $\frac{\partial H_t}{\partial C_t}$ yields the labour supply curve l_t^s:

$$l_t^s = L - \frac{\beta C_t}{(1-\tau_W)\, W_t}$$

- increasing in the wage rate W_t
- decreasing in the labour income tax rate τ_W.

Taking the log of $\frac{\partial H_t}{\partial C_t}$ and substituting it into $\frac{\partial H_t}{\partial K_t}$ yields the optimal consumption path:

$$\frac{\dot{C}_t}{C_t} = R_t - \rho$$

Note that the optimal consumption path is the same as before, whereas the labour supply curve now depends on the after-tax wage rate $(1-\tau_W)\, W_t$

Firm I

There is a representative firm in the economy, and this firm hires labour and rents capital from the household to produce output using the following production function:

$$Y_t = AK_t^{\alpha} l_t^{1-\alpha}$$

- $\alpha \in (0,1)$ is the degree of capital intensity in production
- A is the exogenous level of technology.

The profit function Π_t :

$$\Pi_t = Y_t - R_t K_t - W_t l_t$$

- We have implicitly chosen Y_t as the numeraire (i.e., the price of Y_t is normalised to unity).

Firm II

Differentiating profit function with respect to K_t and l_t :

$$\frac{\partial \Pi_t}{\partial K_t} = \frac{\partial Y_t}{\partial K_t} - R_t = \alpha A \left(\frac{l_t}{K_t} \right)^{1-\alpha} - R_t = 0$$
$$\frac{\partial \Pi_t}{\partial l_t} = \frac{\partial Y_t}{\partial l_t} - W_t = (1-\alpha) A \left(\frac{K_t}{l_t} \right)^{\alpha} - W_t = 0$$

These two equations are the demand functions for K_t and l_t.

Note that the demand functions for K_t and l_t are also the same as before.

The long-run effects of labour income tax I

We start with the long-run effects of labour income tax.
In the long run, the level of capital fully adjusts to its steady-state equilibrium level.
Recall that the optimal consumption path :

$$\frac{\dot{C}_t}{C_t} = R_t - \rho$$

In the steady state, we have $\dot{C}_t = 0$. Therefore, the long-run supply curve of capital is perfectly elastic and given by

$$R_t = \rho$$

The labour supply curve is

$$W_t = \frac{1}{1-\tau_W}\frac{\beta C_t}{L - l_t}.$$

The long-run effects of labour income tax II

The demand curves of capital and labour:

$$R_t = \alpha A \left(\frac{l_t}{K_t}\right)^{1-\alpha} = \alpha \frac{Y_t}{K_t}$$

$$W_t = (1-\alpha) A \left(\frac{K_t}{l_t}\right)^{\alpha} = (1-\alpha) \frac{Y_t}{l_t}$$

Combining labour supply and labour demand:

$$l_t = L - \frac{\beta C_t}{(1-\tau_W)\, W_t} = L - \frac{\beta C_t}{(1-\tau_W)\,(1-\alpha)\, Y_t} l_t$$

The long-run effects of labour income tax III

Given the assumption of a zero capital depreciation rate (i.e., $\delta = 0$), the steady-state equilibrium level of investment I^* is zero.
Therefore, the steady-state equilibrium level of consumption is given by

$$C^* = Y^* - G^* = (1-\gamma)Y^*$$

which is proportional to the steady-state equilibrium level of output.

Substituting C^* into l_t yields the steady-state equilibrium level of labour l^* :

$$l^* = \frac{L}{1 + \frac{\beta}{1-\alpha}\left(\frac{1-\gamma}{1-\tau_W}\right)},$$

which is decreasing in the labour income tax rate τ_W.

The long-run effects of labour income tax IV

- Intuitively, an increase in the labour income tax rate τ_W reduces the after-tax wage rate, which in turn causes the household to substitute leisure for consumption (i.e., a substitution effect) and supply less labour l.
- Graphically, it shifts the labour supply curve to the left; as a result, the equilibrium level of labour l decreases and the pre-tax wage rate W increases (Figure 6.1).
- In the capital market, the decrease in the level of labour shifts the capital demand curve to the left.
- Given the horizontal long-run capital supply curve, the rental price R_t remains at the initial level whereas the equilibrium level of capital decreases in the long run (Figure 6.2)

The long-run effects of labour income tax V

- The decrease in capital shifts the labour demand curve to the left (Figure 6.1). As a result, the pre-tax wage rate W decreases and returns to the initial level because the capital-labour ratio K/l is independent of $\tau_W \cdot$
- Although the pre-tax wage rate W does not change in the long run, the after-tax wage rate $(1 - \tau_W) W$ decreases.
- Finally, the production function $Y_t = AK_t^\alpha l_t^{1-\alpha}$ implies that the decreases in labour and capital both give rise to a decrease in the steady-state equilibrium level of output Y^*.

The long-run effects of labour income tax VI

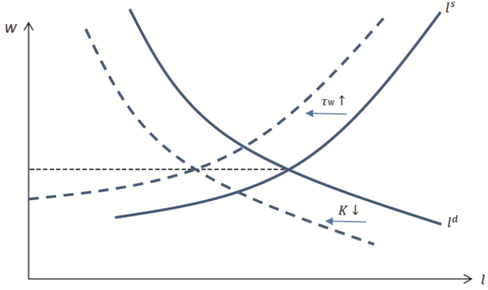

Figure 6.1. Labour market in the long run.

The long-run effects of labour income tax VII

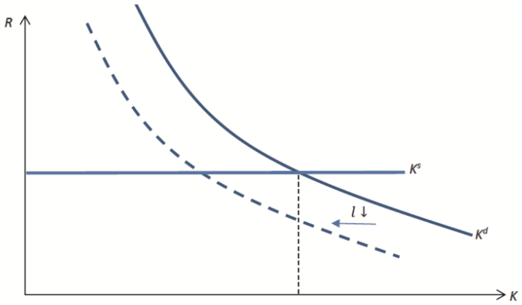

Figure 6.2. Capital market in the long run.

The long-run effects of labour income tax VIII

The long-run effects of labour income tax τ_W :

Long-run effects of an increase in τ_W				
Y	K	R	l	W
decrease	decrease	no change	decrease	no change

Short-Run Effects of Labour Income Tax I

- To complete our analysis, we now look at the short-run effects of labour income tax.
- We define the short run as the moment when the parameter τ_W changes.
- At this moment, the level of capital in the economy is predetermined.
- In other words, the short-run supply curve of capital is vertical.
- As before, an increase in the labour income tax rate τ_W reduces the after-tax wage rate, which in turn causes the household to substitute leisure for consumption (i.e., a substitution effect) and supply less labour.
- Graphically, it shifts the labour supply curve to the left; as a result, the equilibrium level of labour decreases and the pre-tax wage rate W increases. (Figure 6.3)

Short-Run Effects of Labour Income Tax II

- In the capital market, the decrease in the level of labour shifts the capital demand curve to the left.
- Given the vertical short-run capital supply curve, the rental price decreases whereas the equilibrium level of capital remains unchanged in the short run. (Figure 6.4)
- Finally, the production function $Y_t = AK_t^{\alpha} l_t^{1-\alpha}$ implies that the decrease in labour gives rise to a decrease in the level of output.

Short-Run Effects of Labour Income Tax III

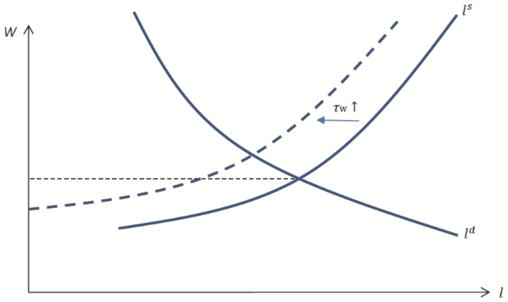

Figure 6.3. Labour market in the short run.

Short-Run Effects of Labour Income Tax IV

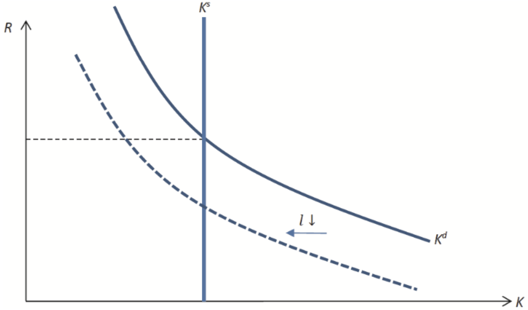

Figure 6.4. Capital market in the short run.

Short-Run Effects of Labour Income Tax V

The short-run effects of labour income tax rate τ_W :

Short-run effects of an increase in τ_W				
Y	K	R	l	W
decrease	no change	decrease	decrease	increase

Government Spending Financed by Labour Income Tax I

What happens when the government uses labour income tax to finance its spending?

In this case, we eliminate lump-sum tax T_t and modify the balanced budget condition as $G_t = \tau_W W_t l_t$, which can be re-expressed as

$$\gamma = (1 - \alpha)\tau_W$$

using $G_t = \gamma Y_t$ and $W_t l_t = (1 - \alpha) Y_t$. In other words, whenever the government increases spending γ, it has to raise the labour income tax rate according to $\tau_W = \gamma / (1 - \alpha)$.

Government Spending Financed by Labour Income Tax II

Substituting this equation into l^* yields

$$l^* = \frac{L}{1 + \beta\left(\frac{1-\gamma}{1-\alpha-\gamma}\right)}$$

which is decreasing in γ unless $\alpha \to 0$.

In other words, the substitution effect of government spending γ via labour income tax τ_W dominates the income effect of government spending (unless the degree of capital intensity α approaches zero, in which case they cancel each other).

As a result, the steady-state equilibrium levels of capital and output are decreasing in γ.

Summary I

- We analyse the effects of permanent changes in the labour income tax rate in the NGM with elastic labour supply.
- We find that by decreasing the after-tax wage rate, an increase in labour income tax gives rise to a substitution effect on labour supply and shifts the labour supply curve to the left.
- In the short run, the equilibrium level of labour decreases and the pre-tax wage rate increases while the level of output decreases.
- The decrease in labour causes a general-equilibrium effect on the capital market by shifting the capital demand curve to the left.
- As a result, the rental price of capital decreases because the short-run capital supply curve is perfectly inelastic.
- Given that the capital supply curve becomes perfectly elastic in the long run, the equilibrium level of capital decreases and in turn affects the labour market by shifting the labour demand curve to the left.

Summary II

- At the end, the equilibrium level of labour decreases by an even larger amount whereas the pre-tax wage rate returns to the initial level.
- In summary, an increase in the labour income tax rate has a contractionary effect on the macroeconomy and decreases the levels of output, capital and labour without affecting the rental price and the pre-tax wage rate in the long run.

Capital Income Tax I

- We considered a labour income tax.
- However, labour income is not the only source of income that is taxed by the government.
- In this part, we consider instead a capital income tax, which is another tax instrument that we commonly observe in reality.
- In this case, we find that capital income tax is also contractionary by decreasing the accumulation of capital.

Household I

The household's utility function:

$$U = \int_0^{\infty} e^{-\rho t} \left[\ln C_t + \beta \ln \left(L - l_t \right) \right] dt$$

- $\rho > 0$ is the household's discount rate
- $\beta > 0$ determines the importance of leisure $L - l_t$ relative to consumption C_t in the utility function.
- l_t is the level of employment chosen by the household.
- The household elastically supplies l_t units of labour to earn a wage income W_t.
- The household accumulates capital K_t and rents it to the representative firm to earn an after-tax capital-rental income $(1 - \tau_R) R_t$, where $\tau_R > 0$ is the tax rate on capital income.

Household II

The asset-accumulation equation is

$$\dot{K}_t = (1 - \tau_R)\, R_t K_t + W_t l_t - C_t - T_t$$

- The capital depreciation rate is zero.
- T_t is a lump-sum tax.

Government I

- The government collects tax revenue to pay for government spending G_t.
- The balanced budget condition is $G_t = T_t + \tau_R R_t K_t$.
- We define the ratio of government spending to output as $\gamma \equiv G_t / Y_t$.
- We are interested in the effects of changes in the capital tax rate τ_R on other macroeconomic variables.
- We previously saw that changes in G_t cause an income effect on the household.
- To separate this income effect from our analysis, we assume that changes in the capital tax rate τ_R are balanced by changes in the lump-sum tax T_t while the government-spending ratio γ does not change.

Hamiltonian I

The Hamiltonian function of the household:

$$H_t = \ln C_t + \beta \ln (L - l_t) + \lambda_t \left[(1 - \tau_R) R_t K_t + W_t l_t - C_t - T_t \right].$$

The first-order conditions:

$$\frac{\partial H_t}{\partial l_t} = -\frac{\beta}{L - l_t} + \lambda_t W_t = 0$$
$$\frac{\partial H_t}{\partial C_t} = \frac{1}{C_t} - \lambda_t = 0$$
$$\frac{\partial H_t}{\partial K_t} = \lambda_t (1 - \tau_R) R_t = \lambda_t \rho - \dot{\lambda}_t$$

Recall that K_t is a state variable (i.e., a variable that accumulates over time), so we have to set $\partial H_t / \partial K_t = \lambda_t \rho - \dot{\lambda}_t$.

Hamiltonian II

Combining $\frac{\partial H_t}{\partial l_t}$ and $\frac{\partial H_t}{\partial C_t}$ yields the labour supply curve l_t^s :

$$l_t^s = L - \frac{\beta C_t}{W_t}$$

- increasing in the wage rate W_t (i.e., a substitution effect)
- decreasing in consumption C_t (i.e., an income effect).

Taking the log of $\frac{\partial H_t}{\partial C_t}$ and substituting it into $\frac{\partial H_t}{\partial K_t}$ yields the optimal consumption path:

$$\frac{\dot{C}_t}{C_t} = (1 - \tau_R)\, R_t - \rho,$$

which now depends on the after-tax rental price $(1 - \tau_R)\, R_t$.

Firm I

There is a representative firm in the economy, and this firm hires labour and rents capital from the household to produce output using the following production function:

$$Y_t = AK_t^{\alpha} l_t^{1-\alpha},$$

- $\alpha \in (0,1)$ is the degree of capital intensity in production
- A is the exogenous level of technology.

The profit function Π_t is

$$\Pi_t = Y_t - R_t K_t - W_t l_t,$$

* We have implicitly chosen Y_t as the numeraire (i.e., the price of Y_t is normalised to unity).

Firm II

Differentiating profit function with respect to K_t and l_t :

$$\frac{\partial \Pi_t}{\partial K_t} = \frac{\partial Y_t}{\partial K_t} - R_t = \alpha A \left(\frac{l_t}{K_t} \right)^{1-\alpha} - R_t = 0$$
$$\frac{\partial \Pi_t}{\partial l_t} = \frac{\partial Y_t}{\partial l_t} - W_t = (1-\alpha) A \left(\frac{K_t}{l_t} \right)^{\alpha} - W_t = 0$$

These two equations are the demand functions for K_t and l_t.
Note that the demand functions for K_t and l_t are also the same as before.

Long-Run Effects of Capital Income Tax I

$$\frac{\dot{C}_t}{C_t} = (1 - \tau_R) R_t - \rho$$

shows that capital income tax affects the household's optimal consumption path, which in turn determines the supply of capital.

However, because the short-run capital supply curve is perfectly inelastic, capital income tax has no effect on the equilibrium level of capital at the moment when the tax rate τ_R changes.

Therefore, we focus on the long-run effects of capital income tax in our analysis.

Long-Run Effects of Capital Income Tax II

In the long run, the level of capital fully adjusts to its steady-state equilibrium level. Recall that the optimal consumption path is given by

$$\frac{\dot{C}_t}{C_t} = (1 - \tau_R) R_t - \rho$$

In the steady state, we have $\dot{C}_t = 0$. Therefore, the long-run supply curve of capital is perfectly elastic:

$$R_t = \frac{\rho}{1 - \tau_R},$$

The labour supply curve:

$$W_t = \frac{\beta C_t}{L - l_t}.$$

Long-Run Effects of Capital Income Tax III

The demand curves of capital and labour:

$$R_t = \alpha A \left(\frac{l_t}{K_t}\right)^{1-\alpha} = \alpha \frac{Y_t}{K_t},$$
$$W_t = (1-\alpha) A \left(\frac{K_t}{l_t}\right)^{\alpha} = (1-\alpha) \frac{Y_t}{l_t}.$$

- An increase in the capital income tax rate τ_R reduces the aftertax capital rental price and causes the household to accumulate less capital.
- Graphically, it shifts the capital supply upwards; as a result, the equilibrium level of capital K decreases in the capital market (Figure 7.2).
- In the labour market, the decrease in the level of capital shifts the labour demand curve to the left.

Long-Run Effects of Capital Income Tax IV

- However, the labour supply curve shifts to the right (due to a negative income effect from the reduction in output Y and consumption C) to completely offset the shift in the labour demand curve such that the equilibrium level of labour remains at the initial level. (Figure 7.1)

Long-Run Effects of Capital Income Tax V

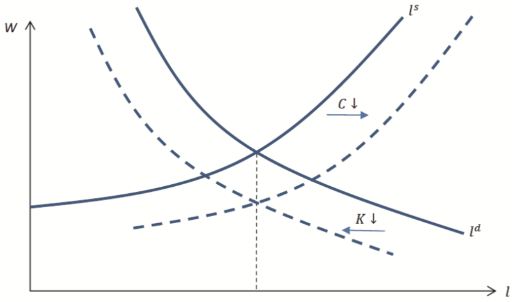

Figure 7.1. Labour market in the long run.

Long-Run Effects of Capital Income Tax VI

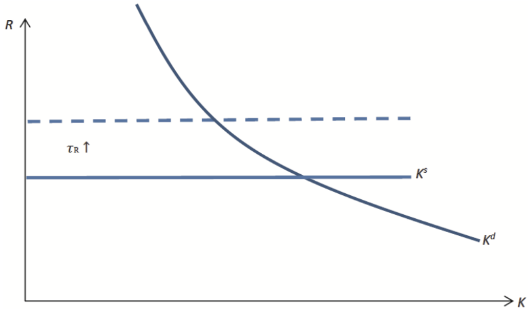

Figure 7.2. Capital market in the long run.

Long-Run Effects of Capital Income Tax VII

To derive the steady-state equilibrium level of labour, we combine labour supply and labour demand to obtain

$$l_t = L - \frac{\beta C_t}{W_t} = L - \frac{\beta C_t}{(1-\alpha) Y_t} l_t.$$

Given the assumption of a zero capital depreciation rate (i.e., $\delta = 0$), the steady-state equilibrium level of investment I^* is zero. Therefore, the steady-state equilibrium level of consumption is given by

$$C^* = Y^* - G^* = (1-\gamma) Y^*,$$

which is proportional to the steady-state equilibrium level of output.

Long-Run Effects of Capital Income Tax VIII

Substituting C^* into l_t yields the steady-state equilibrium level of labour l^* given by

$$l^* = \frac{L}{1 + \beta(1-\gamma)/(1-\alpha)},$$

which is independent of the capital income tax rate τ_R.

In other words, changes in the capital tax rate τ_R do not affect the steadystate equilibrium level of labour l^* (in the absence of capital depreciation).

Finally, the production function $Y_t = AK_t^{\alpha} l_t^{1-\alpha}$ implies that the decrease in capital gives rise to a decrease in the steady-state equilibrium level of output Y^*.

Long-Run Effects of Capital Income Tax IX

The long-run effects of capital income tax τ_R :

Long-run effects of an increase in τ_R				
Y	K	R	l	W
decrease	decrease	increase	no change	decrease

Government Spending Financed by Capital Income Tax I

What happens when the government uses capital income tax to finance its spending?

In this case, we eliminate lump-sum tax T_t and modify the balanced budget condition as $G_t = \tau_R R_t K_t$, which can be re-expressed as

$$\gamma = \alpha \tau_R,$$

using $G_t = \gamma Y_t$ and $R_t K_t = \alpha Y_t$

In other words, whenever the government increases spending γ, it has to raise the capital income tax rate according to $\tau_R = \gamma / \alpha$.

Given that the capital income tax rate τ_R does not appear in l^*, an increase in government spending γ increases the steady-state equilibrium level of labour.

Government Spending Financed by Capital Income Tax II

However, the resulting increase in the capital income tax rate τ_R leads to a decrease in the steady-state equilibrium level of capital as in Figure 7.2.

Therefore, when government spending is financed by capital income tax, its overall effect on the steady-state equilibrium level of output is ambiguous.

Summary I

- We analyse the long-run effects of permanent changes in the capital income tax rate in the neoclassical growth model with elastic labour supply.
- We find that an increase in the capital income tax rate causes the household to accumulate less capital and reduces the steady-state equilibrium level of capital, which in turn has a general-equilibrium effect on the labour market by shifting the labour demand curve to the left and by depressing the wage rate and the equilibrium level of labour.
- The decrease in the levels of capital and output reduces the level of consumption and gives rise to an income effect, which shifts the labour supply curve to the right.

Summary II

- As a result, the wage rate decreases by a larger amount and the equilibrium level of labour may return to the initial level (depending on the capital depreciation rate).
- In summary, an increase in the capital income tax rate has a contractionary effect on the macroeconomy by decreasing the wage rate and the levels of output, capital and generally labour but raises the pre-tax rental price of capital such that the after-tax rental price remains unchanged in the long run.

New Keynesian Model I

- Now, we consider monetary policy.
- Recall that in the neoclassical growth model, prices are fully flexible.
- In this case, changes in the level of money supply do not have any effect on real variables (i.e., the neutrality of money).
- Therefore, we need to introduce sticky prices into our model.
- However, before we can consider sticky prices, we need to first develop a model in which firms have price-setting power.
- In other words, firms need to have the power to set their prices before they can decide whether or not to change their prices.
- Consequently, we need to convert the market structure from perfect competition to monopolistic competition.
- In summary, we develop a New Keynesian model and find that increasing the money supply has an expansionary effect on the macroeconomy in the short run by increasing the demand for goods.

A Simple New Keynesian Model I

- Given the complexity of the firm side, we keep the household side as simple as possible.
- Specifically, the household has an upward-sloping labour supply curve and a perfectly inelastic capital supply curve in the short run.
- We focus our analysis on the short run because sticky prices are a short-run phenomenon and monetary policy only has short-run effects.
- In the long run, prices become fully flexible, and the effects of monetary policy become neutral.

A Simple New Keynesian Model II

On the firm side, we need to distinguish between competitive firms that produce a final good and monopolistic firms that produce intermediate goods.

There are N monopolistic firms that are indexed by $i \in [1, N]$ and sell differentiated intermediate goods y_i.

The production function of monopolistic firm i :

$$y_i = AK_i^{\alpha} l_i^{1-\alpha},$$

where $\alpha \in (0, 1)$ and $\{K_i, l_i\}$ are capital and labour employed by firm i.

A Simple New Keynesian Model III

There is also a representative firm that produces final output Y by combining the differentiated intermediate goods using the following production function:

$$Y = \left(\sum_{i=1}^{N} y_i^{\varepsilon} \right)^{1/\varepsilon}$$

which is known as a constant elasticity of substitution (CES) aggregator in which the parameter $\varepsilon \in (0, 1)$ determines the substitution elasticity $1/(1-\varepsilon)$ between the differentiated intermediate goods.

As ε approaches unity, the substitution elasticity $1/(1-\varepsilon)$ goes to infinity, in which case the intermediate goods become perfect substitutes.

In other words, the degree of substitutability between products is increasing in ε.

The less substitutable the products are (i.e., a smaller ε), the more market power the monopolistic firms have.

Final Output I

A representative firm produces final good, and the profit function Π

$$\Pi = PY - \sum_{i=1}^{N} p_i y_i = P\left(\sum_{i=1}^{N} y_i^{\varepsilon}\right)^{1/\varepsilon} - \sum_{i=1}^{N} p_i y_i$$

- P is the price of final good Y
- p_i is the price of intermediate good y_i.

The market structure in this sector is perfectly competitive, and the firm takes the prices as given.

Final Output II

The first-order condition with respect to y_i :

$$\frac{\partial \Pi}{\partial y_i} = \frac{1}{\varepsilon} P \left(\sum_{i=1}^{N} y_i^{\varepsilon} \right)^{\frac{1-\varepsilon}{\varepsilon}} \varepsilon y_i^{\varepsilon - 1} - p_i = 0$$

which can be expressed as

$$p_i = PY^{1-\varepsilon} y_i^{\varepsilon-1} \Leftrightarrow y_i^d = \left(\frac{P}{p_i} \right)^{1/(1-\varepsilon)} Y$$

This is the demand function y_i^d, in which the demand elasticity is $1/(1-\varepsilon)$. As ε approaches unity, the demand elasticity $1/(1-\varepsilon)$ goes to infinity, in which case the demand curve for y_i becomes perfectly elastic.

Intermediate Goods I

A monopolistic firm produces intermediate product i.
The profit function π_i :

$$\pi_i = p_i y_i - Wl_i - RK_i$$

- Here, we make the following assumption to simplify our analysis: the level of capital supplied to each firm is fixed in the short run.
- Under this assumption, firm i can only change its labour input whenever y_i changes in the short run.
- The market structure in this sector is monopolistically competitive, so that the firm has a price-setting power.
- In other words, the firm sets its own price p_i, instead of taking it as given.

Intermediate Goods II

Substituting the demand function into the profit function

$$\pi_i = PY^{1-\varepsilon} y_i^{\varepsilon} - Wl_i - RK_i,$$

- $\{W, R\}$ are the wage rate and the capital rental price as before.

Differentiating profit function with respect to l_i yields

$$\frac{\partial \pi_i}{\partial l_i} = \varepsilon \underbrace{PY^{1-\varepsilon} y_i^{\varepsilon-1}}_{=p_i} \frac{\partial y_i}{\partial l_i} - W = 0,$$

which can be re-expressed as firm i 's labour demand:

$$W = \varepsilon p_i MPL_i \Leftrightarrow l_i^d = \frac{\varepsilon(1-\alpha)p_i y_i}{W},$$

where $\varepsilon < 1$.

Intermediate Goods III

In other words, a profit-maximising monopolistic firm would set its value of marginal product of labour above the wage rate (i.e., $W < p_i MPL_i$).

$W = \varepsilon p_i MPL_i$ can also be re-expressed as

$$p_i = \frac{1}{\varepsilon}\frac{W}{MPL_i} = \frac{1}{\varepsilon}MC_i,$$

- W/MPL_i is the marginal cost MC_i of production.
- Given that $\varepsilon < 1$, the monopolistic price is above the marginal cost of production (i.e., $p_i > MC_i$).
- The markup ratio $1/\varepsilon > 1$ implies that the monopolistic firm makes a positive profit.

Intermediate Goods IV

The positive monopolistic profit enables the firm to allow its price to temporarily deviate from its profit-maximising level without making a loss.

For example, p_i equation shows that when the wage rate W increases, the firm would want to raise its price p_i to maximise profit. However, there

may be some frictions (e.g., a menu cost) that prevent an immediate price adjustment. We summarise these frictions as sticky prices. When prices

are sticky, monopolistic firms may not be able to maximise profit, but they would continue production so long as $p_i > MC_i$.

Short-Run Effects of Monetary Policy I

In this section, we explore the short-run effects of monetary policy.

We define the short run as the duration in which the prices of firms do not change.

Recall that the demand function y_i^d is given by

$$y_i^d = \left(\frac{P}{p_i}\right)^{1/(1-\varepsilon)} Y$$

which is decreasing in the relative price p_i/P and increasing in aggregate output Y.

To relate aggregate output to the level of money supply in the economy, we introduce the quantity theory of money given by

$$MV = PY$$

- M is the level of money supply.

Short-Run Effects of Monetary Policy II

- V is the velocity of money in the economy.
- For simplicity, we set $V = 1$.

Substituting $MV = PY$ into y_i^d yields

$$y_i^d = \left(\frac{P}{p_i}\right)^{1/(1-\varepsilon)} \frac{M}{P},$$

which relates the demand function y_i^d to the level of money supply M. Given the assumption of sticky prices p_i, the aggregate price level P is also fixed in the short run.

Therefore, an increase in the level of money supply M would increase the demand y_i^d for product $i \in [1, N]$ by increasing aggregate output Y.

Short-Run Effects of Monetary Policy III

Suppose the level of money supply M increases by a small amount given by $\Delta M > 0$. Then, the increase in the demand y_i^d is

$$\Delta y_i^d = \left(\frac{P}{p_i}\right)^{1/(1-\varepsilon)} \frac{\Delta M}{P}.$$

To satisfy the increased demand for its product, firm $i \in [1, N]$ needs to employ more labour, and the increase in the labour demand l_i^d is

$$\Delta l_i^d = \frac{\Delta y_i^d}{MPL_i} = \frac{1}{(1-\alpha)A\left(K_i/l_i\right)^{\alpha}} \left(\frac{P}{p_i}\right)^{1/(1-\varepsilon)} \frac{\Delta M}{P},$$

where we have used the definition of the marginal product of labour $MPL_i \equiv \Delta y_i / \Delta l_i$.

* $P = \left[\sum_{i=1}^{N} p_i^{\varepsilon/(\varepsilon-1)}\right]^{(\varepsilon-1)/\varepsilon}$.

Short-Run Effects of Monetary Policy IV

Given that all firms $i \in [1, N]$ demand more labour, aggregate labour demand increases by $\sum_{i=1}^{N} \Delta l_i^d$.

Graphically, the labour demand curve shifts to the right. In the labour market, the equilibrium level of labour l and the real wage rate W/P increase (Figure 8.1).

Short-Run Effects of Monetary Policy V

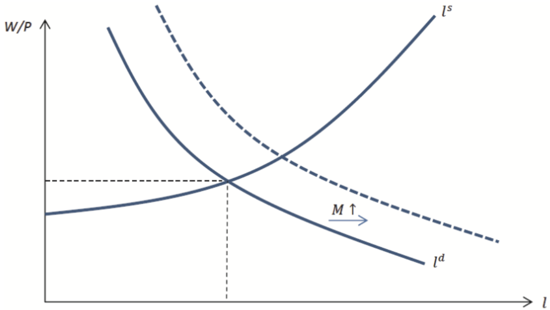

Figure 8.1. Labour market in the short run.

Short-Run Effects of Monetary Policy VI

The short-run effects of an increase in money supply M :

Short-run effects of an increase in M			
Y	l	W/P	W
increase	increase	increase	increase

Long-Run Effects of Monetary Policy I

- An increase in the level of money supply has an expansionary effect on the economy, but only in the short run.
- When prices become fully flexible in the long run, the price level P increases by the same proportion as the level of money supply M.
- Then, the expansionary effect disappears, and the real variables $\{Y, I, W/P\}$ return to their initial levels.
- Therefore, in the long run, an increase in the level of money supply M only causes the nominal variables $\{P, W, R\}$ to increase by the same proportion without affecting the real variables.
- This result is known as the neutrality of money and the classical dichotomy.

Summary I

- We analyse the effects of monetary policy in a New Keynesian model.
- To allow for sticky prices, we consider a monopolistically competitive product market in which monopolistic firms have price-setting power.
- This price-setting power enables each firm to price its differentiated product above the marginal cost of production.
- The presence of this markup allows the firm to let its price temporarily deviate from the profit-maximising level while still making a positive monopolistic profit.
- When prices are sticky in the short run, an increase in the level of money supply increases the demand for goods, which in turn increases the demand for factor inputs (e.g., labour).
- In this case, an increase in the level of money supply has an expansionary effect on the economy by increasing the level of output, the level of labour, the real wage rate and the nominal wage rate.

Summary II

- However, this expansionary effect disappears when prices fully adjust in the long run, in which case the higher level of money supply increases the price level and the nominal wage rate without affecting the level of output, the level of labour and the real wage rate.