Neoclassical Growth Model

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Macroeconomic Theory

1. A Dynamic General Equilibrium Model
   - Dynamics in the Neoclassical Growth Model
   - Real Business Cycle Model

Notes I

- We focused on the steady state of the neoclassical growth model.
- We only looked at the long-run effects of changes in the level of technology.
- Now, we use a graphical approach to demonstrate the short-run effects of technology, which is a potential source of economic fluctuations in business cycles.
- We also graphically demonstrate the long-run effects of technology for a comparison with its short-run effects.
- This analysis enables us to compare the effects of changes in the level of technology on the macroeconomy at different time horizons.
- In summary, we find that the short-run and long-run supply curves of capital are drastically different, which in turn have interesting implications on the macroeconomic effects of technology.

Short-Run Effects of Technology I

We define the short run as the moment when a parameter (e.g., the level of technology A) changes.

At this moment, the level of capital K_t in the economy is predetermined and cannot be changed immediately.

In other words, the short-run supply curve of capital is vertical.

The assumption of perfectly inelastic supply of labour ($L_t = L$ for all t) implies that the labour supply curve is also vertical.

Short-Run Effects of Technology II

Demand curves of capital and labour:

$$R_t = \alpha A \left(\frac{L_t}{K_t} \right)^{1-\alpha}$$

$$W_t = (1-\alpha) A \left(\frac{K_t}{L_t} \right)^{\alpha}$$

- The rental price equals R_t to the marginal product of capital
- The wage rate equals W_t to the marginal product of labour.
- An increase in the level of technology A shifts the demand curves of capital and labour to the right.
- As a result, the wage rate W_t and the rental price R_t go up.

Short-Run Effects of Technology III

However, given the vertical supply curves of capital and labour, the levels of capital and labour do not change in the short run.
(see Figures 3.1 and 3.2)

Then, the production function

$$Y_t = AK_t^{\alpha} L_t^{1-\alpha}$$

- an increase in technology A gives rise to an increase in the level of output Y_t despite the fact that capital and labour do not change in the short run.

Short-Run Effects of Technology IV

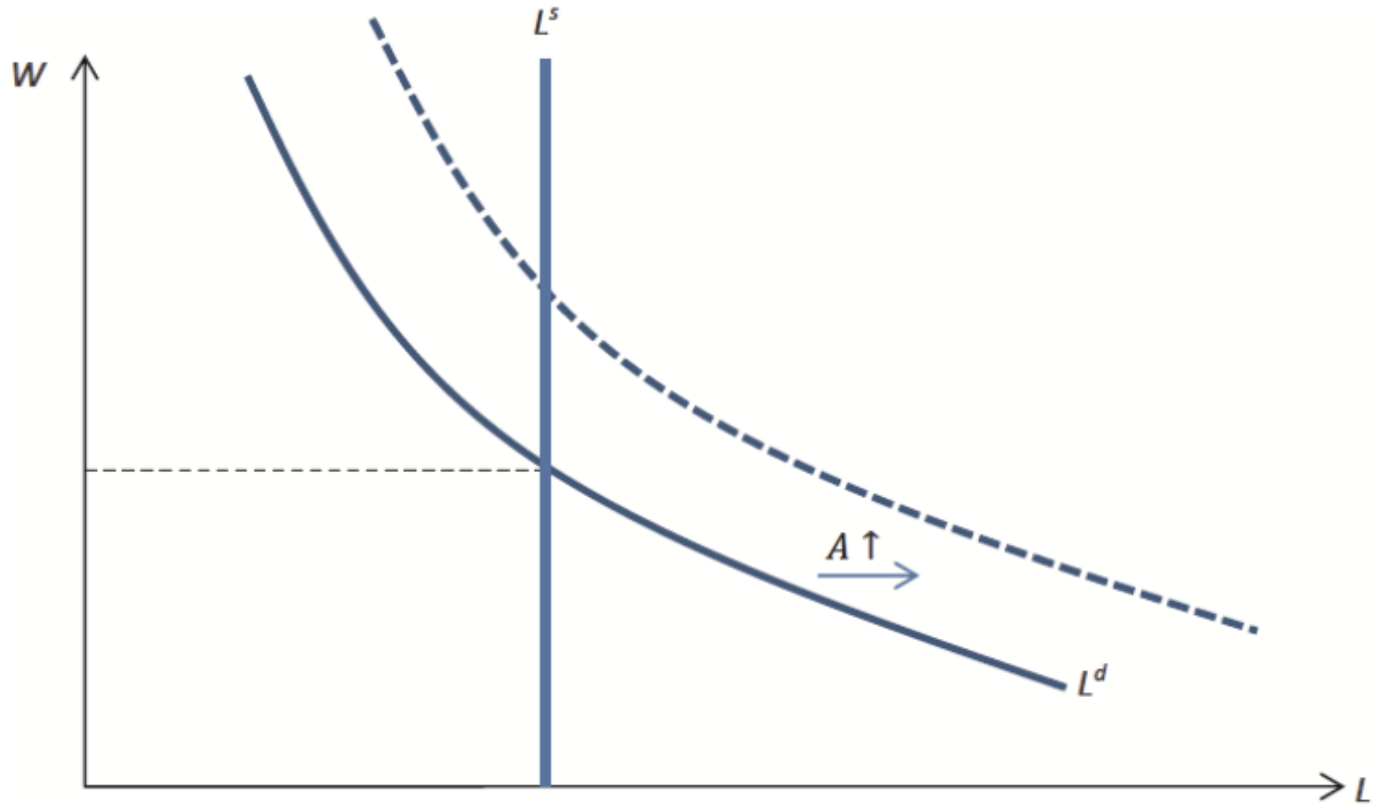

Figure 3.1. Labour market in the short run.

Short-Run Effects of Technology V

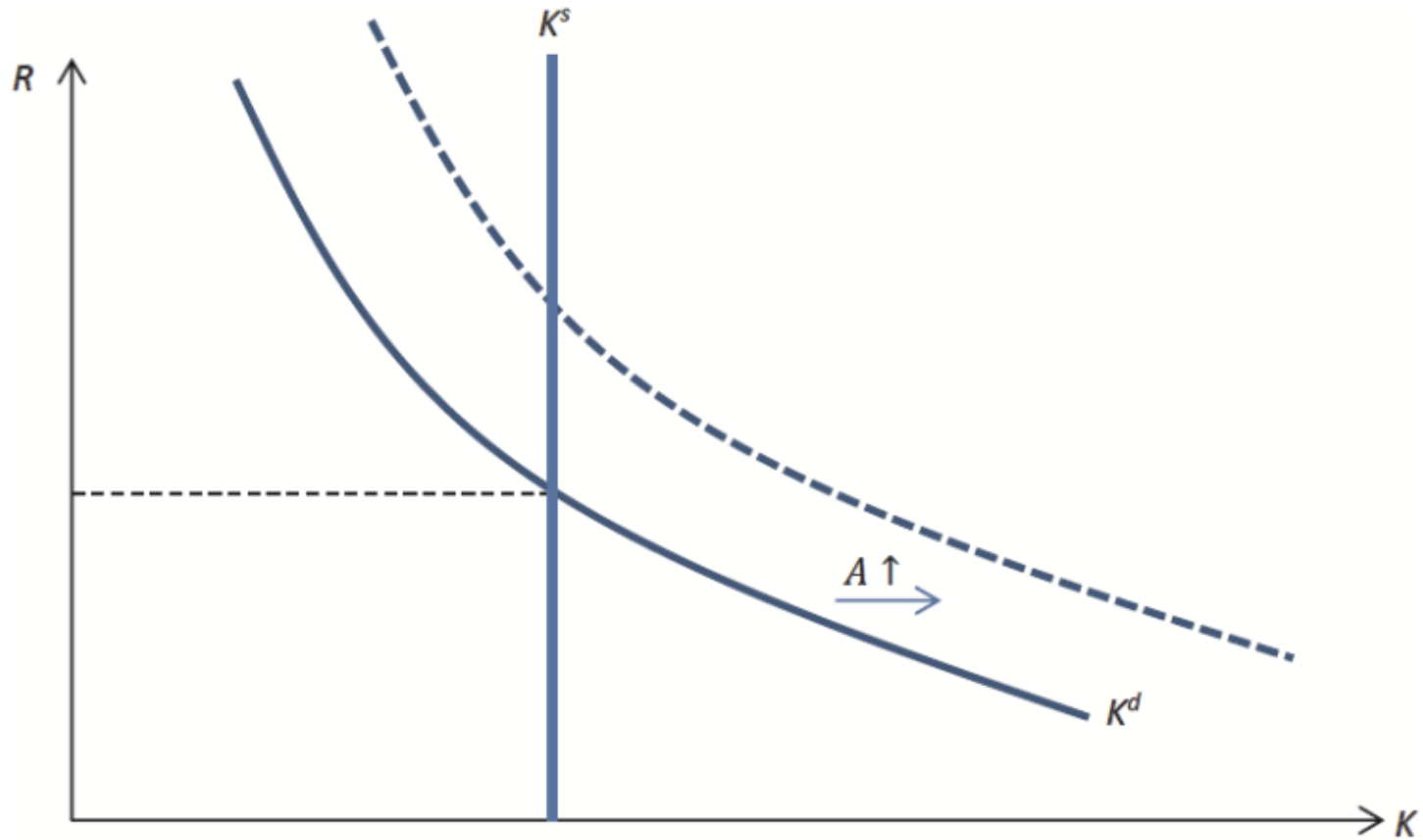

Figure 3.2. Capital market in the short run.

Short-Run Effects of Technology VI

The result:

Short-run effects of an increase in A				
Y	K	R	L	W
increase	no change	increase	no change	increase

Long-Run Effects of Technology I

In the long run, the level of capital fully adjusts to its steady-state equilibrium level.

So, what does the long-run supply curve of capital look like?

Recall that the optimal consumption path derived from the household's utility maximisation is given by

$$\frac{\dot{C}_t}{C_t} = R_t - \rho$$

where the parameter $\rho > 0$ is the household's discount rate.

In the steady state, we have $\dot{C}_t = 0$. Therefore, the steady-state version of the optimal consumption path is given by

$$R_t = \rho$$

Long-Run Effects of Technology II

$$R_t = \rho$$

This equation gives us a horizontal long-run supply curve of capital. In other words, the long-run supply curve of capital is perfectly elastic.

The increase in the level of technology A has shifted the demand curves of capital and labour to the right.

The horizontal long-run supply curve of capital implies that the rental price returns to the initial level whereas the equilibrium level of capital increases in the long run; see Figure 3.4.

The increase in the equilibrium level of capital has an additional positive effect on the wage rate by shifting the labour demand curve further to the right in the long run; see Figure 3.3.

Long-Run Effects of Technology III

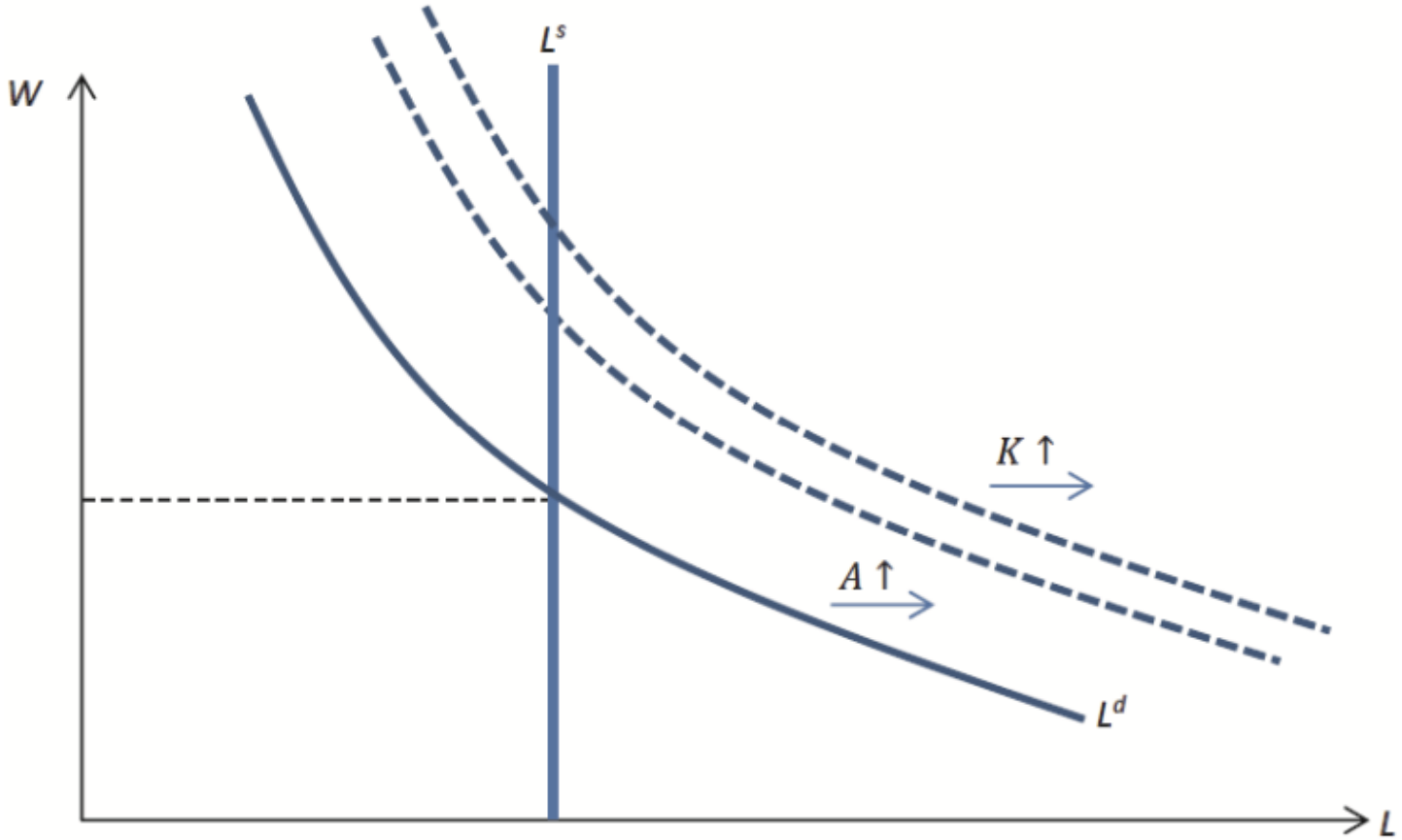

Figure 3.3. Labour market in the long run.

Long-Run Effects of Technology IV

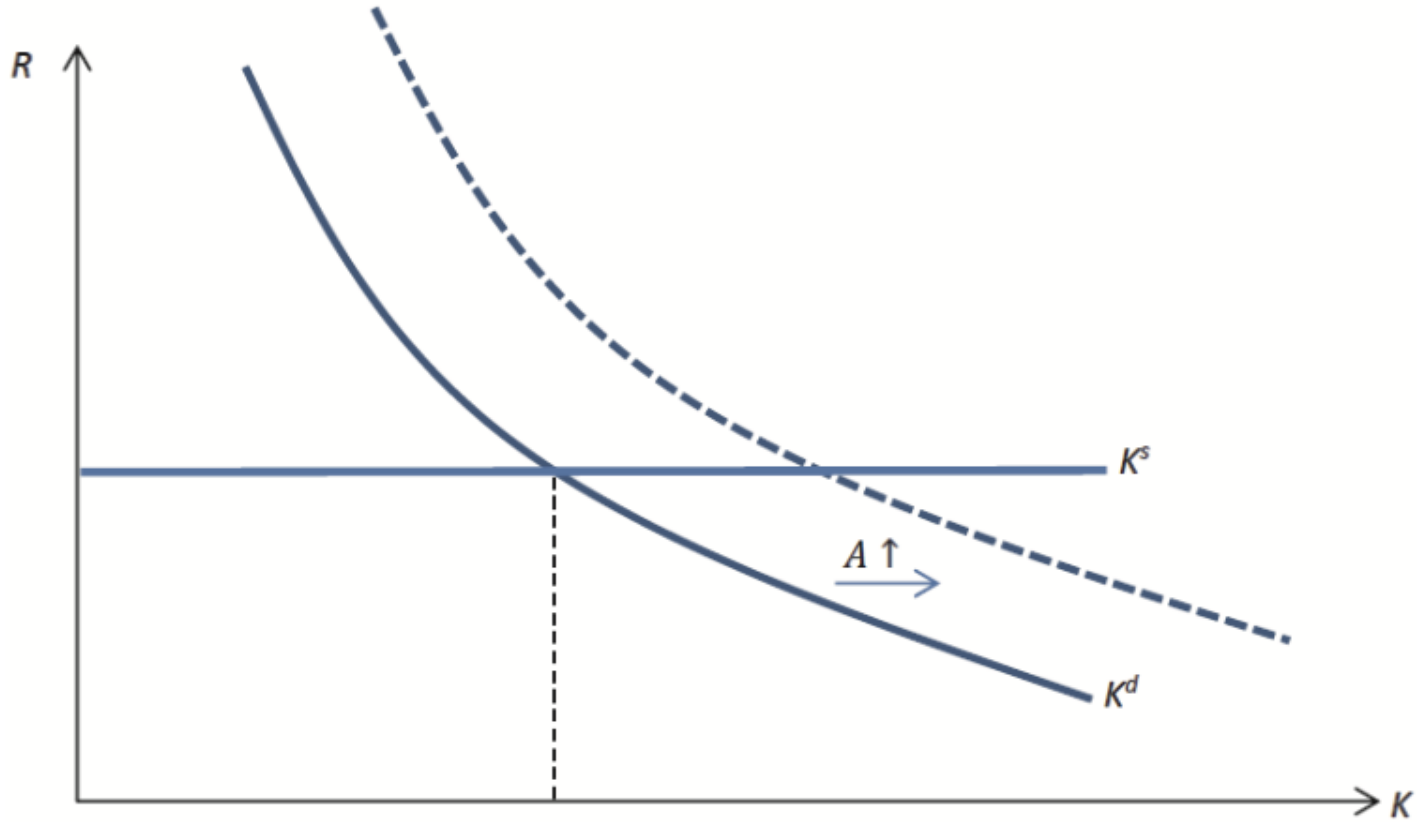

Figure 3.4. Capital market in the long run.

Long-Run Effects of Technology V

The production function $Y_t = AK_t^{\alpha} L_t^{1-\alpha}$ implies that the increases in technology A and capital K both give rise to an increase in the steady-state equilibrium level of output Y^*.

Furthermore, the increase in the steady-state equilibrium level of capital K^* implies that the long-run increases in the level of output and the wage rate are larger than their short-run increases.

The long-run effects of technology A can be summarised as follows:

Long-run effects of an increase in A				
Y	K	R	l	W
increase	increase	no change	no change	increase

Dynamics I

We can also predict what happens as the economy moves from the short run to the long run.

After the level of technology A increases, the level of output Y_t increases immediately whereas the level of capital K_t increases gradually, which leads to a further gradual increase in output Y_t; see Figures 3.5 and 3.6.

Dynamics II

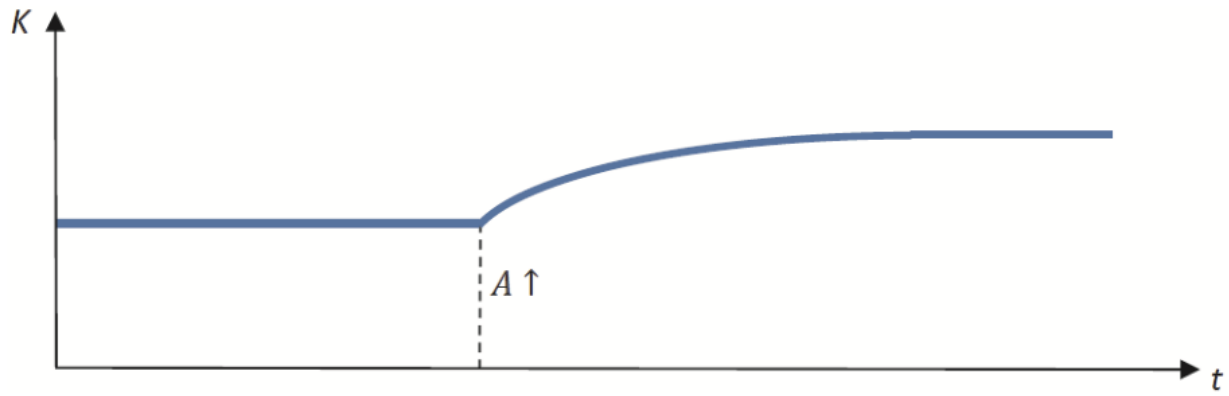

Figure 3.5. Time path of capital.

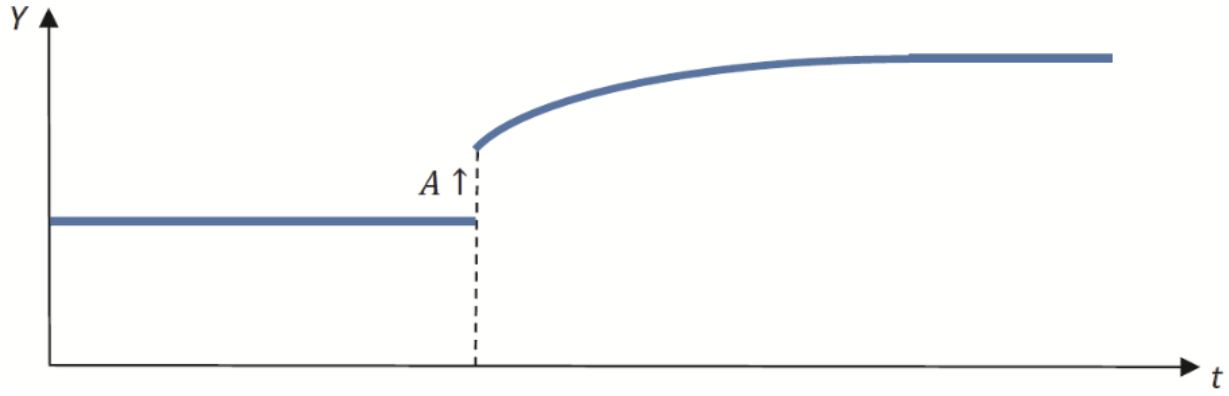

Figure 3.6. Time path of output.

Dynamics III

As we move along the capital demand curve, the rental price R_t gradually decreases towards the initial level; see Figure 3.7.

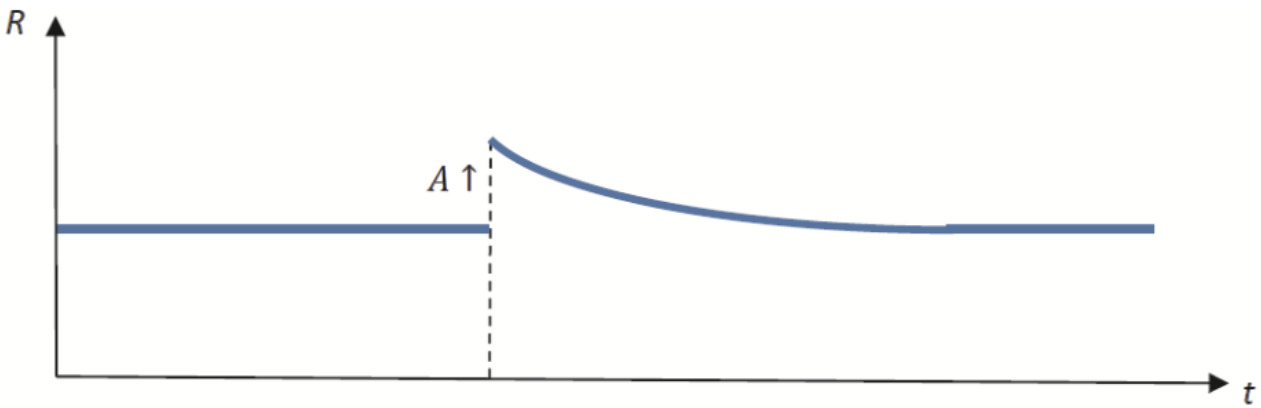

Figure 3.7. Time path of rental price.

Dynamics IV

The increase in the level of capital gradually shifts the labour demand curve further to the right. As we move along the labour supply curve, the wage rate W_t increases further and gradually converges towards the new steady-state equilibrium level; see Figure 3.8.

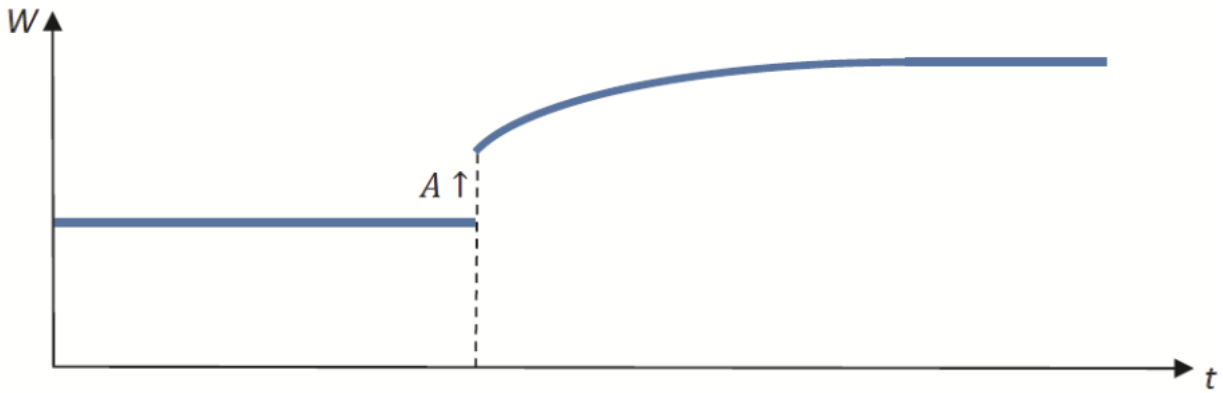

Figure 3.8. Time path of wage rate.

Summary I

- We use a graphical approach to examine the effects of technology in the neoclassical growth model.
- We find that the effects of permanent changes in the level of technology vary across time because the short-run and long-run supply curves of capital are very different.
- In the short run, the capital supply curve is perfectly inelastic, so that changes in the level of technology do not affect the equilibrium level of capital.
- In this case, an increase in the level of technology increases the level of output, the wage rate and the rental price without affecting the equilibrium levels of labour and capital.

Summary II

- In the long run, the capital supply curve becomes perfectly elastic, so that an increase in the level of technology raises the equilibrium level of capital, which in turn has a general-equilibrium effect on the labour market by shifting the labour demand curve further to the right and causing a larger increase in the wage rate.
- In this case, an increase in the level of technology increases the level of output, the wage rate and the equilibrium level of capital without affecting the rental price and the equilibrium level of labour (due to the assumption of perfectly inelastic labour supply).
- Finally, an unrealistic implication of the neoclassical growth model is that the level of labour (i.e., employment) never changes.

Intro

- We know that the level of labour (i.e., employment) never changes in the neoclassical growth model with perfectly inelastic labour supply.
- Now, we generalise the neoclassical growth model to allow for elastic labour supply chosen by the utility-maximising household.
- In this framework, the supply of labour is determined by a substitution effect and an income effect, which are both influenced by changes in technology.
- Therefore, this modification of elastic labour supply allows for fluctuations in employment, which are an important feature of business cycles.
- The neoclassical growth model with elastic labour supply is essentially a special case of the real business cycle (RBC) model.

Households I

We introduce the choice of leisure into the household's utility function:

$$U = \int_0^{\infty} e^{-\rho t} \left[\ln C_t + \beta \ln \left(L - l_t\right)\right] dt$$

- The parameter $\rho > 0$ is the household's discount rate
- The parameter $\beta > 0$ determines the importance of leisure $L - l_t$ relative to consumption C_t in the utility function
- L is the household's total labour endowment, and l_t is the level of employment chosen by the household.
- The household elastically supplies l_t units of labour to earn a wage income W_t.
- It accumulates capital K_t and rents it to the representative firm to earn a capital-rental income R_t.

Households II

The asset-accumulation equation is modified as follows:

$$\dot{K}_t = R_t K_t + W_t l_t - C_t,$$

we assume a zero depreciation rate of capital (i.e., $\delta = 0$).

In general, the asset-accumulation equation is given by
$\dot{K}_t = (R_t - \delta) K_t + W_t l_t - C_t$

Households III

The Hamiltonian function is given by

$$H = lnC_t + \beta ln(L - l_t) + \lambda_t(R_t K_t + W_t l_t - C_t)$$

FOCs

$$\frac{\partial H_t}{\partial l_t} = -\frac{\beta}{L - l_t} + \lambda_t W_t = 0$$
$$\frac{\partial H_t}{\partial C_t} = \frac{1}{C_t} - \lambda_t = 0$$
$$\frac{\partial H_t}{\partial K_t} = \lambda_t R_t = \lambda_t \rho - \dot{\lambda}_t$$

Recall that K_t is a state variable (i.e., a variable that accumulates over time), so we have to set $\partial H_t / \partial K_t = \lambda_t \rho - \dot{\lambda}_t$.

Households IV

Combining $\frac{\partial H_t}{\partial l_t}$ and $\frac{\partial H_t}{\partial C_t}$ yields the labour supply curve l_t^s given by

$$l_t^s = L - \frac{\beta C_t}{W_t},$$

which is increasing in the wage rate W_t (i.e., a substitution effect) and decreasing in consumption C_t (i.e., an income effect).

- A higher wage rate increases the opportunity cost of leisure and causes the household to supply more labour (a substitution effect)
- A higher level of consumption decreases the marginal utility of consumption from wage income and causes the household to enjoy more leisure (an income effect)

Households V

- If leisure is not important to the household (i.e., $\beta = 0$), then we have $l_t^s = L$, in which case we are back to the case of perfectly inelastic labour supply.
- If leisure matters to the household (i.e., $\beta > 0$), then unemployment $L - l_t^s$ is positive.

Households VI

Taking the log of $\frac{\partial H_t}{\partial C_t}$ and substituting it into $\frac{\partial H_t}{\partial K_t}$ yields the optimal consumption path:

$$\frac{\dot{C}_t}{C_t} = R_t - \rho$$

Firm I

The firm's optimisation problem is the same as before.
There is a representative firm in the economy, and this firm hires labour and rents capital from the household to produce output using the following production function:

$$Y_t = AK_t^{\alpha} l_t^{1-\alpha},$$

where the parameter $\alpha \in (0, 1)$ is the degree of capital intensity in production and A is the exogenous level of technology.

The profit function Π_t is

$$\Pi_t = Y_t - R_t K_t - W_t l_t,$$

where we have chosen Y_t as the numeraire (i.e., the price of Y_t is normalised to unity).

Firm II

Differentiating the profit function with respect to K_t and l_t yields

$$\frac{\partial \Pi_t}{\partial K_t} = \frac{\partial Y_t}{\partial K_t} - R_t = \alpha A \left(\frac{l_t}{K_t} \right)^{1-\alpha} - R_t = 0$$
$$\frac{\partial \Pi_t}{\partial l_t} = \frac{\partial Y_t}{\partial l_t} - W_t = (1-\alpha) A \left(\frac{K_t}{l_t} \right)^{\alpha} - W_t = 0$$

These two equations are the demand functions for K_t and l_t.

Short-Run Effects of Technology I

Once again, we define the short run as the moment when a parameter (e.g., the level of technology A) changes.
At this moment, the level of capital in the economy is predetermined. In other words, the short-run supply curve of capital is vertical as before.
Howewer, we have an upward-sloping labour supply curve given by

$$W_t = \frac{\beta C_t}{L - l_t}$$

As for the demand curves of labour and capital, they are given by

$$R_t = \alpha A \left(\frac{l_t}{K_t}\right)^{1-\alpha} = \alpha \frac{Y_t}{K_t}$$

$$W_t = (1-\alpha) A \left(\frac{K_t}{l_t}\right)^{\alpha} = (1-\alpha) \frac{Y_t}{l_t}$$

Short-Run Effects of Technology II

- An increase in the level of technology A shifts the demand curves of labour and capital to the right.
- In the labour market, the wage rate W_t and the equilibrium level of labour l_t increase.
- Therefore, elastic labour supply gives rise to a positive effect of technology A on employment l_t
- They have a general-equilibrium effect on the capital market.
- In the capital market, the rental price R_t increases, whereas the equilibrium level of capital does not change in the short run;

see Figures 4.1 and 4.2.

Short-Run Effects of Technology III

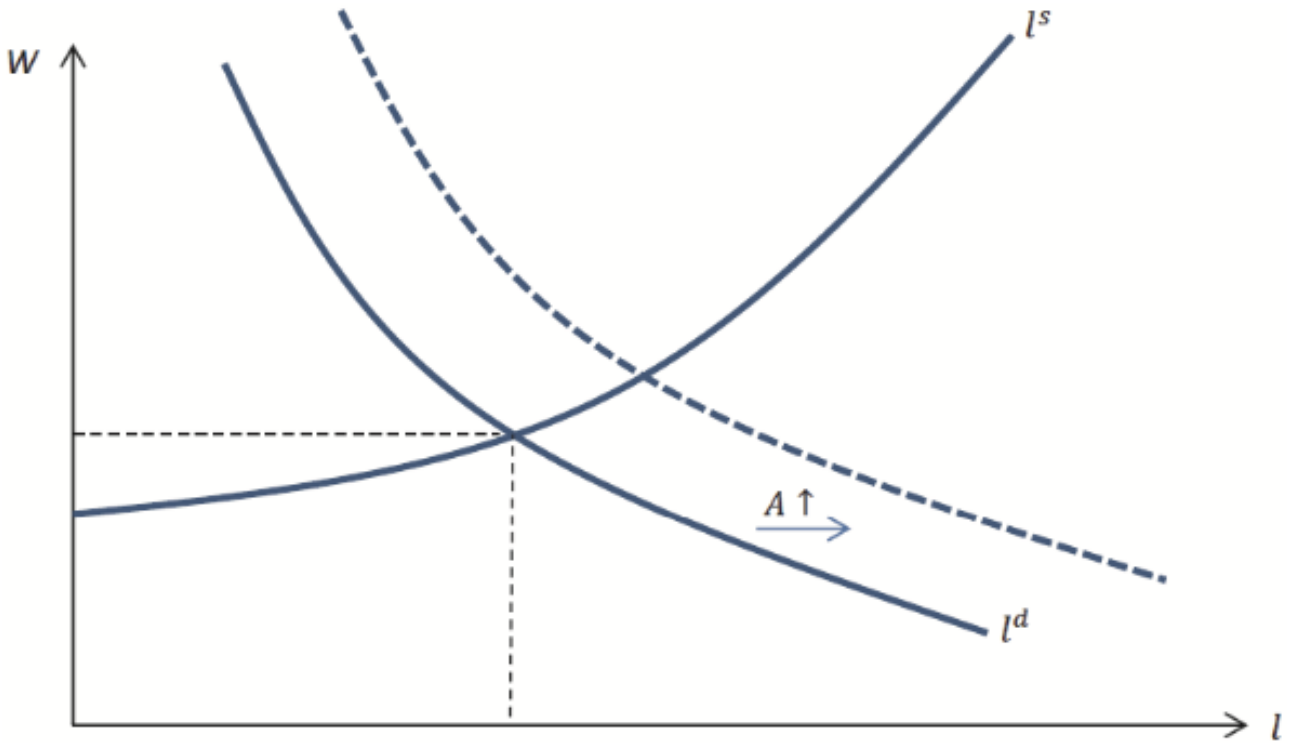

Figure 4.1. Labour market in the short run.

Short-Run Effects of Technology IV

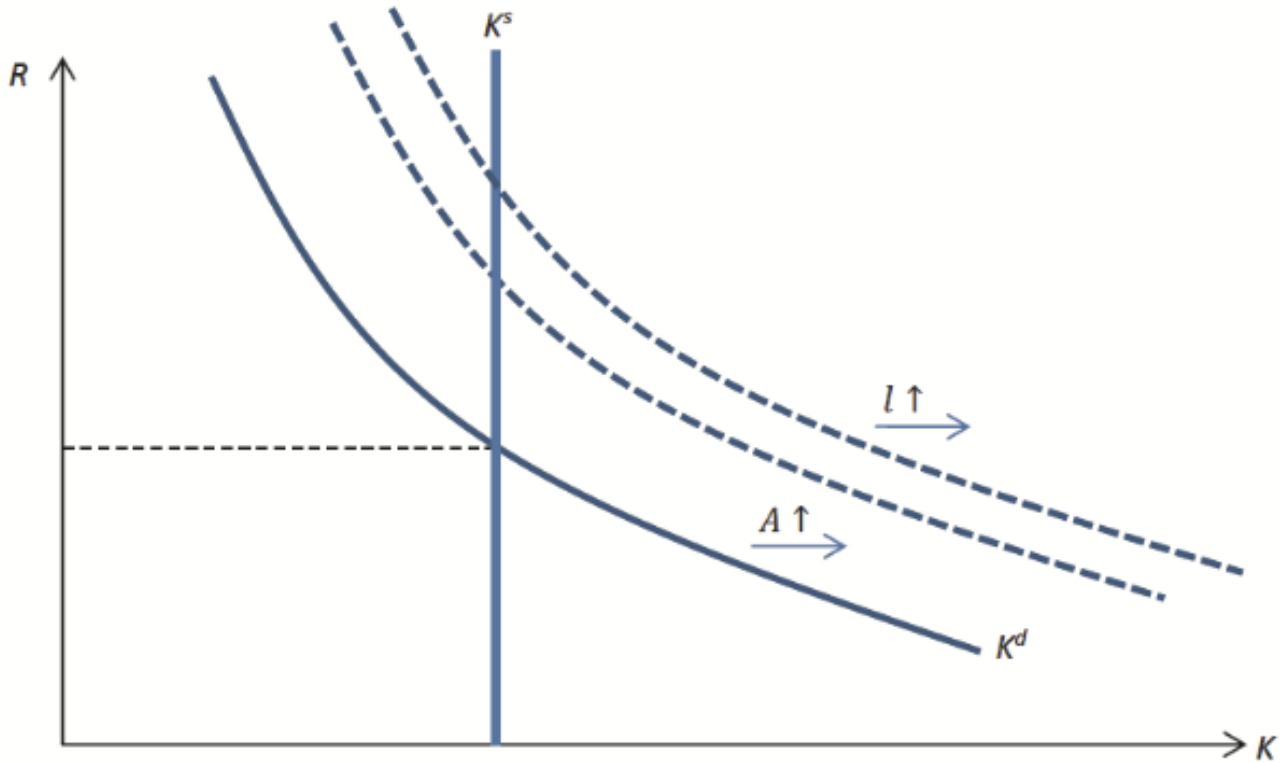

Figure 4.2. Capital market in the short run.

Short-Run Effects of Technology V

Then, the production function

$$Y_t = AK_t^{\alpha} l_t^{1-\alpha}$$

implies that the increases in technology A and labour l_t both give rise to an increase in the level of output Y_t.

The short-run effects of technology A can be summarised as follows:

Short-run effects of an increase in A				
Y	K	R	l	W
increase	no change	increase	increase	increase

Long-Run Effects of Technology I

In the long run, the level of capital fully adjusts to its steady-state equilibrium level K^*.

Recall that the optimal consumption path is given by

$$\frac{\dot{C}_t}{C_t} = R_t - \rho$$

In the steady state, we have $\dot{C}_t = 0$.
Therefore, the long-run supply curve of capital is perfectly elastic and given by

$$R_t = \rho$$

Long-Run Effects of Technology II

- The increase in the level of technology A shifts the demand curves of capital and labour to the right.
- In the capital market, the rental price R_t returns to the initial level whereas the equilibrium level of capital increases in the long run; see Figure 4.4.
- In the labour market, we now allow the labour supply curve to shift. As can be seen in Figure 4.3, the labour supply curve shifts to the left (due to an increase in the level of consumption) and completely offsets the shift in the labour demand curve.
- Therefore, the equilibrium level of labour l_t returns to the initial level.
- However, there continues to be a positive effect on the wage rate W_t in the long run; see Figure 4.3.

Long-Run Effects of Technology III

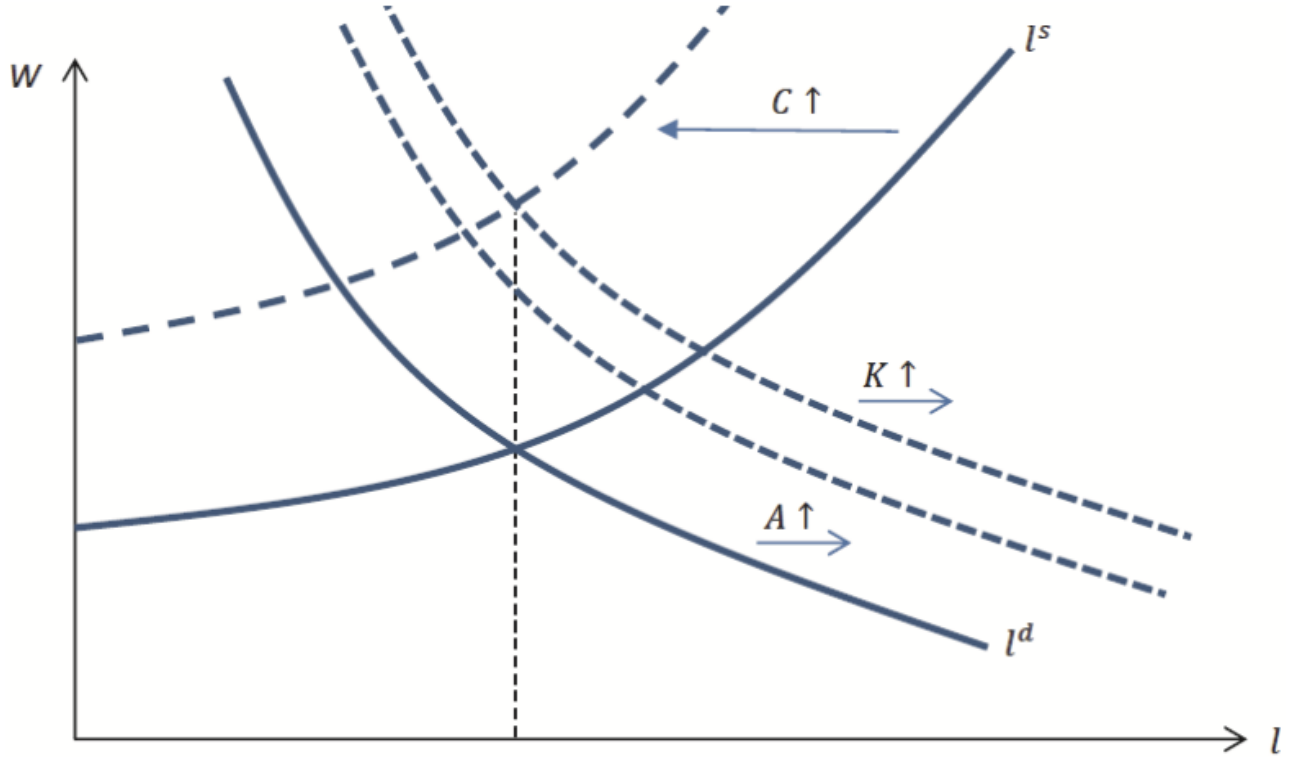

Figure 4.3. Labour market in the long run.

Long-Run Effects of Technology IV

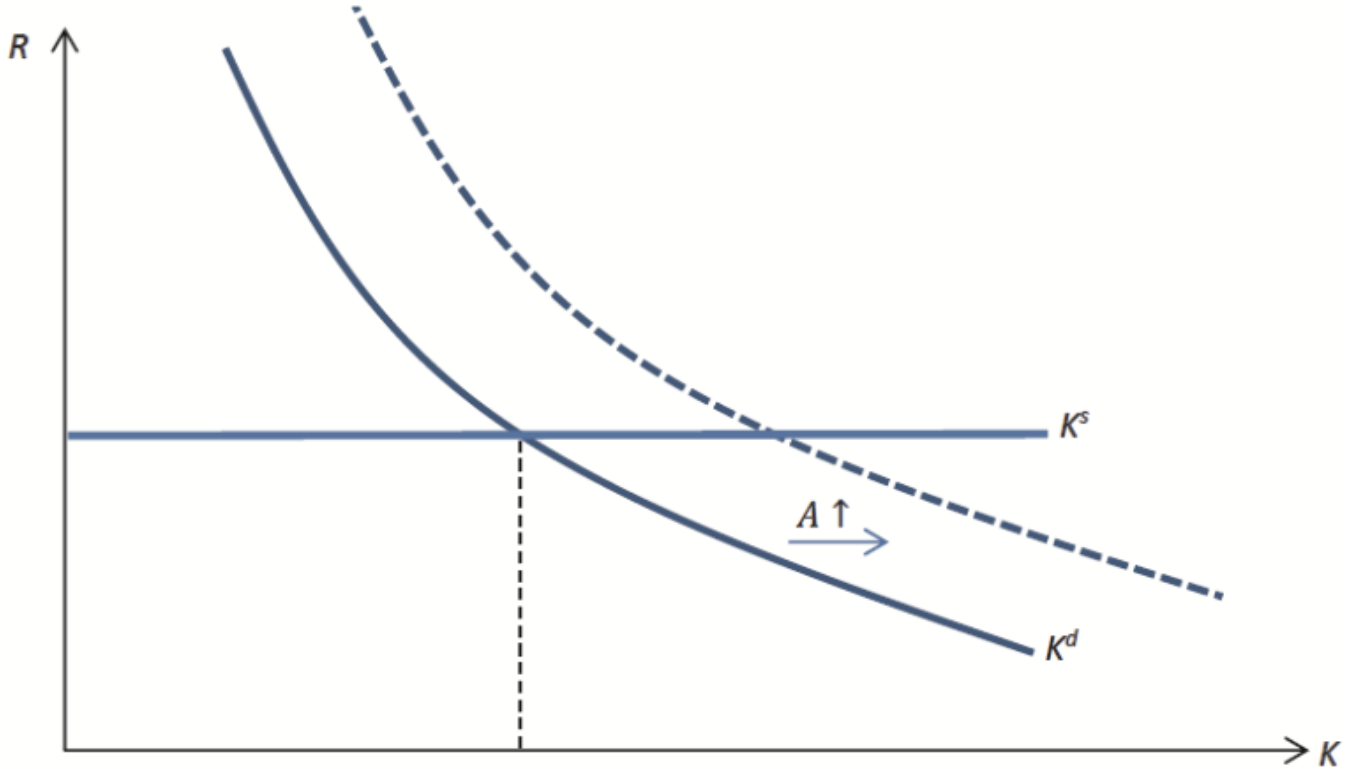

Figure 4.4. Capital market in the long run.

Long-Run Effects of Technology V

To see that the steady-state equilibrium level of labour returns to the initial level, we substitute $W_t = (1-\alpha)A\left(\frac{K_t}{l_t}\right)^{\alpha} = (1-\alpha)\frac{Y_t}{l_t}$ into $l_t^s = L - \frac{\beta C_t}{W_t}$ to derive

$$l_t = L - \frac{\beta C_t}{W_t} = L - \frac{\beta C_t}{(1-\alpha)Y_t} l_t.$$

Given the assumption of a zero capital depreciation rate (i.e., $\delta = 0$), the steady-state equilibrium level of investment I^* is zero.

Therefore, the steady-state equilibrium levels of consumption and output are the same such that $C^* = Y^*$.

Long-Run Effects of Technology VI

The steadystate equilibrium level of labour l^* is given by

$$l^* = \frac{L}{1 + \beta/(1-\alpha)}$$

which is independent of technology A.

- Changes in the level of technology A do not affect the steady-state equilibrium level of labour l^*.
- The steady-state equilibrium level of labour l^* is decreasing in leisure preference β and capital intensity α.
- If leisure becomes more important (i.e., a larger β) to the household, it would supply less labour to the labour market.
- If labour becomes less important in production (i.e., a larger α), the firm would demand less labour in the labour market.

Long-Run Effects of Technology VII

Finally, the production function $Y_t = AK_t^{\alpha} l_t^{1-\alpha}$ implies that the increases in technology A and capital K both give rise to an increase in the steady-state equilibrium level of output Y^*.

Furthermore, the increase in the steady-state equilibrium level of capital K^* and the leftward shift in the labour supply curve imply that the long-run increase in the wage rate is larger than its short-run increase.

The long-run effects of technology A can be summarised as follows:

Long-run effects of an increase in A				
Y	K	R	l	W
increase	increase	no change	no change	increase

Dynamics of the Model I

$\Rightarrow$ Given the additional adjustment in labour, the dynamics becomes more complicated in the neoclassical growth model with elastic labour supply, but we can still conjecture what happens as the economy moves from the short run to the long run.

- After the level of technology A increases, the level of output Y_t increases immediately whereas the level of capital K_t increases gradually.
- As we move along the capital demand curve, the rental price R_t gradually decreases towards the initial level.
- The increase in the level of capital gradually shifts the labour demand curve further to the right.
- Simultaneously, the labour supply curve gradually shifts to the left.

Dynamics of the Model II

- These shifts in the labour demand and supply curves give rise to a gradual increase in the wage rate W_t towards the new steady-state equilibrium level and a gradual decrease in the level of labour l_t towards the initial level.

Summary I

- We extend the neoclassical growth model by allowing for elastic labour supply.
- In the model, the representative household chooses leisure in addition to consumption and saving.
- Maximising the household's utility, we derive the labour supply curve.
- The household's supply of labour is increasing in the wage rate (a substitution effect) and decreasing in the level of consumption (an income effect)
- Given the upward-sloping labour supply curve, an increase in the level of technology shifts the labour demand curve to the right and increases the equilibrium level of labour in the short run.
- The change in level of labour in has a general-equilibrium effect on the capital market by shifting the capital demand curve further to the right and causing a larger increase in the rental price.

Summary II

- In this case, an increase in the level of technology increases the level of out-put, the level of labour, the wage rate and the rental price without affecting the level of capital in the short run.
- In the long run, the higher level of consumption gives rise to an income effect on labour supply and shifts the labour supply curve to the left.
- As a result, the equilibrium level of labour returns to the initial level. However, the long-run capital supply curve becomes perfectly elastic, and the equilibrium level of capital increases in the long run.
- In this case, an increase in the level of technology increases the level of output, the level of capital and the wage rate without affecting the rental price and the level of labour in the long run.