Rational Expectations and Economic Policy

Macroeconomic Theory

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Outline

Aims of this lecture

- What do we mean by the Rational Expectations Hypothesis (REH)?
- What are the implications of the REH for the conduct of economic policy? The "Policy-Ineffectiveness Proposition" (PIP)
- What are the implications of the REH for economic modelling? The "Lucas critique"?
- What is "real" and what is "gimmick" about the way the REH was sold to the economics profession?

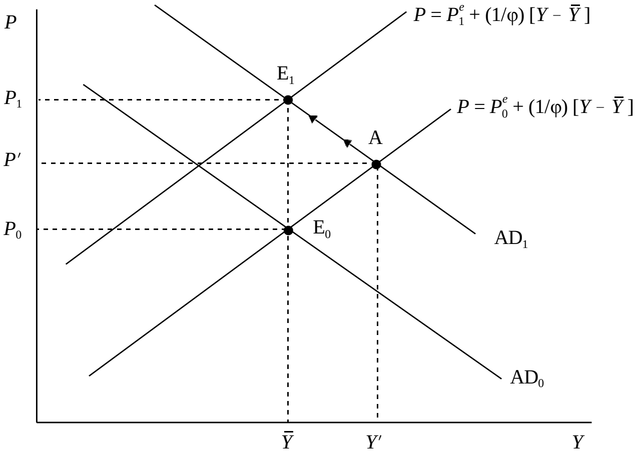
Reminder

- Recall how policy works in a neoclassical synthesis model with AEH:

$$
\begin{aligned}
Y &= AD(\underset{+}{G}, \underset{+}{M/P}), \qquad && AD_G > 0, \quad AD_{M/P} > 0 \\
Y &= Y^* + \phi \left[P - P^e \right], && \phi > 0 \\
\dot{P}^e &= \lambda \left[P - P^e \right], && \lambda > 0
\end{aligned}
$$

- Initially in full equilibrium in point E_0 ($Y = Y^*$, $P = P_0 = P_0^e$) (see **Figure 5.1**)
  - Increase in the money supply shifts the AD curve out
  - Initial effect: move from E_0 to point A
  - In point A: expectations falsified ($P' \neq P_0 = P_0^e$)
  - Gradually over time the economy moves back to the new full equilibrium in E_1 (where $Y = Y^*$, $P = P_1 = P_1^e$)

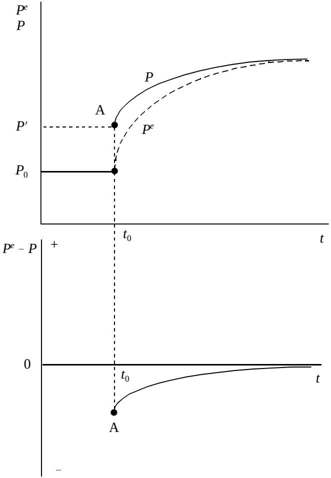
Figure 5.1: Monetary policy under adaptive expectations



Observation

- **Odd adjustment path under the AEH**: economics is based on the assumption of rational agents
- But, as **Figure 5.2** shows, under the AEH agents make **systematic** mistakes along the entire adjustment path
- In the present case all errors are negative, i.e. there is systematic underestimation of the price level $(P^e < P)$ during the adjustment period

Figure 5.2: Expectational errors under adaptive expectations



Reaction

- This prompted John Muth to postulate the REH
- Rational agents do not waste scarce resources (of which information is one)!
- REH in words: subjective expectation (P_t^e) coincides with the objective expectation conditional on the information set of the agent

Simple example of a market for some agricultural good

- Assume that the market for this good is captured by the following equations:

$$
\begin{aligned}
Q_t^D &= a_0 - a_1 P_t, && a_1 > 0 \\
Q_t^S &= b_0 + b_1 P_t^e + U_t, && b_1 > 0 \\
Q_t^D &= Q_t^S \quad [\equiv Q_t]
\end{aligned}
$$

  - Demand depends on actual price in current period
  - Supply depends on **expectation** regarding the current price (takes time to raise a pig!)
  - Supply is subject to stochastic shocks, U_t (weather, swine fever)
  - The market clears and demand equals supply

Information set

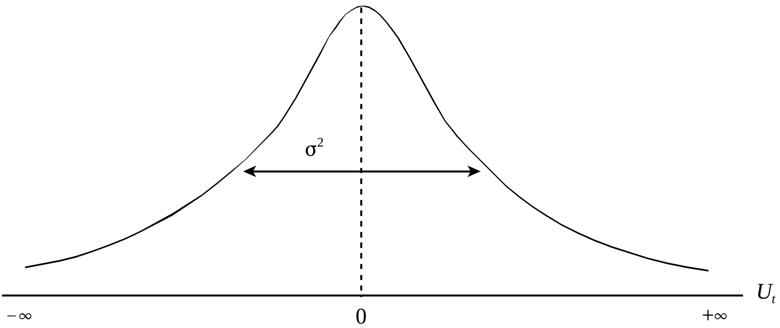
- Information set available when the supply decision is made (period $t-1$) is denoted by Ω_{t-1}:

$$\Omega_{t-1} \equiv \{\underbrace{P_{t-1}, P_{t-2}, ...; Q_{t-1}, Q_{t-2}, ...}_{(a)}; \underbrace{a_0, a_1, b_0, b_1}_{(b)}; \underbrace{U_t \sim N(0, \sigma^2)}_{(c)}\}$$

  (a) Agents do not forget (relevant) past information
  (b) Agents know the parameters of the model
  (c) Agents know the stochastic process of the shocks (e.g. the normal distribution, as is drawn in **Figure 5.3**. Can be any distribution.)

- REH in mathematical form: $P_t^e = E\left(P_t \mid \Omega_{t-1}\right) \equiv E_{t-1}P_t$, where we use the shorthand notation E_{t-1} to indicate that the expectation is conditional upon information set Ω_{t-1}

Figure 5.3: The normal distribution



How do we solve this model?

- *Executive summary*: solve the model for its market equilibrium, take expectations, and think, think...!
- The recipe is as follows
- Demand equals supply equals quantity traded:

$$Q_t = a_0 - a_1 P_t = b_0 + b_1 P_t^e + U_t \quad \Longrightarrow$$

$$P_t = \frac{a_0 - b_0 - b_1 P_t^e - U_t}{a_1} \qquad \text{(S1)}$$

How do we solve this model?

- Take expectations based on the information set Ω_{t-1}:

$$E_{t-1}P_t = E_{t-1}\left(\frac{a_0 - b_0 - b_1 P_t^e - U_t}{a_1}\right)$$
$$= \underbrace{\frac{a_0 - b_0}{a_1}}_{(a)} - \underbrace{\frac{b_1}{a_1}}_{(a)}\underbrace{E_{t-1}P_t^e}_{(b)} - \underbrace{\frac{1}{a_1}}_{(a)}\underbrace{E_{t-1}U_t}_{(c)}$$

(a) Take out of expectations operator because a_0, a_1, b_0, and b_1 are in Ω_{t-1}

(b) Expectation of a constant equals that constant, i.e. $E_{t-1}P_t^e = P_t^e$

(c) As $U_t \sim N(0, \sigma^2)$ there is no better prediction than $E_{t-1}U_t = 0$

How do we solve this model?

- We are left with:

$$\underbrace{E_{t-1}P_t}_{(a)} = \frac{a_0 - b_0}{a_1} - \frac{b_1}{a_1}\underbrace{P_t^e}_{(b)} \qquad \text{(S2)}$$

- According to the REH, the objective expectation of the price level ((a) on the left-hand side) must be equal to the subjective expectation by the agents ((b) on the right-hand side). Hence, (S2) can be solved for P_t^e:

$$P_t^e = \frac{a_0 - b_0}{a_1} - \frac{b_1}{a_1}P_t^e \;\Rightarrow$$

$$P_t^e = E_{t-1}P_t = \frac{a_0 - b_0}{a_1 + b_1} \qquad \text{(S3)}$$

Test your understanding

****** Self Test ******

In Chapter 1 we argued that the perfect foresight hypothesis (PFH) is the deterministic counterpart to the REH. Can you see how our agricultural model would be solved under PFH? Show that you will arrive at (S3)?

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Features of the market clearing price level

- What does the actual market clearing price level look like? Substitute P_t^e in the quasi reduced form equation for P_t (see (S1))

$$
\begin{aligned}
P_t &= \frac{1}{a_1}\left[a_0 - b_0 - b_1 \frac{a_0 - b_0}{a_1 + b_1} - U_t\right] \\
&= \frac{a_0 - b_0}{a_1 + b_1} - \frac{1}{a_1} U_t \\
&= \bar{P} - \frac{1}{a_1} U_t
\end{aligned}
$$

where $\bar{P}$ is the equilibrium price that would obtain if there were no stochastic elements in the market (here is the answer to the self test)

Features of the market clearing price level

- The actual market clearing price is stochastic but the best prediction of it (the rational expectation for P_t) is the deterministic equilibrium price in this case
  - See **Figure 5.4** for a computer-generated illustration. Computer generates time series of (quasi-) random numbers
  - In **Figure 5.5** we illustrate how actual and expected price would fluctuate under the AEH

Test your understanding

****** Self Test ******

What would happen to P_t^e *and* P_t *if the supply shock,* U_t*, is autocorrelated, e.g.* $U_t = \rho_U U_{t-1} + \varepsilon_t$ *with* $|\rho_U| < 1$ *and* $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$*?*

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Figure 5.4: Actual and expected price under REH

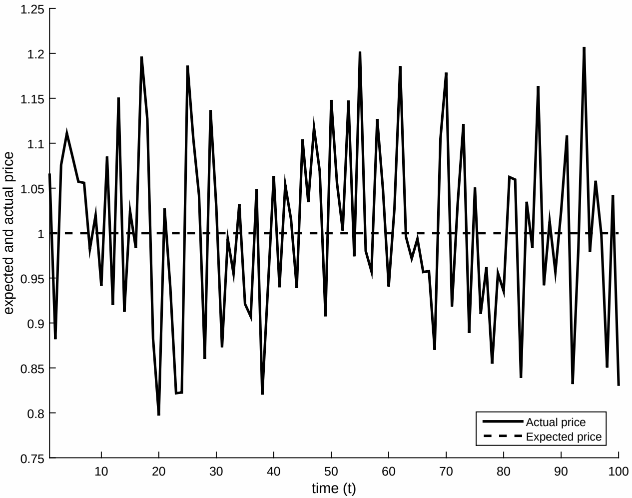
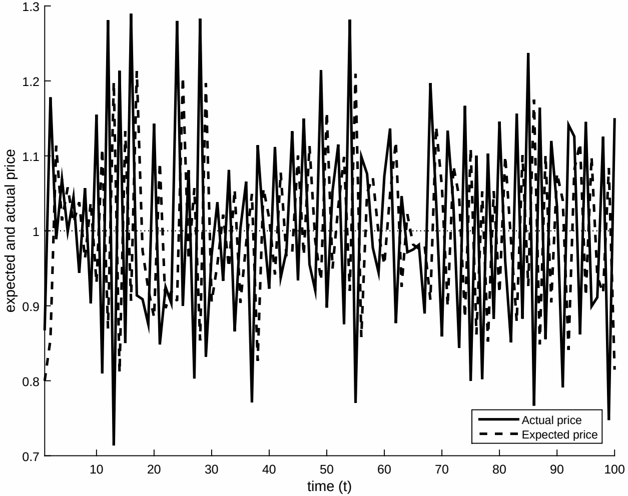


Figure 5.5: Actual and expected price under AEH



Applications of the REH to macroeconomics

- New Classical economists like Lucas, Sargent, Wallace, and Barro introduced the REH into macroeconomics
- Simple IS-LM-AS model with rational expectations:

$$y_t = \alpha_0 + \alpha_1(p_t - E_{t-1}p_t) + u_t \qquad \text{(AS)}$$

$$y_t = \beta_0 + \beta_1(m_t - p_t) + \beta_2 E_{t-1}(p_{t+1} - p_t) + v_t \qquad \text{(AD)}$$

$$m_t = \mu_0 + \mu_1 m_{t-1} + \mu_2 y_{t-1} + e_t \qquad \text{(MSR)}$$

  - All variables are in logarithms, e.g. $y_t \equiv \ln Y_t$ etcetera
  - AS is the aggregate supply curve, $\alpha_1 > 0$, and $u_t \sim N(0, \sigma_u^2)$ is the stochastic shock hitting aggregate supply
  - AD is the aggregate demand curve, $\beta_1, \beta_2 > 0$, and $v_t \sim N(0, \sigma_v^2)$ is the stochastic shock hitting aggregate demand

Applications of the REH to macroeconomics

- Model features (continued).
  - Used approximation $\ln(P_{t+1}/P_t) \approx (P_{t+1}/P_t) - 1$.
  - Expected inflation, $E_{t-1}(p_{t+1} - p_t)$, enters the AD curve because money demand (and thus the LM curve) depends on the nominal interest rate whilst investment demand (and thus the IS curve) depends on the real interest rate ("Tobin effect")
  - MSR is the money supply rule, and $e_t \sim N(0, \sigma_e^2)$ is the stochastic shock in the rule (impossible to perfectly control the money supply)
  - Both u_t and v_t are not autocorrelated

Two key tasks:

- What is the rational expectation solution of the model?
- The variable of most interest, from a stabilization point of view, is (the logarithm of) aggregate output, y_t
- *Can* the policy maker stabilize the economy by choosing the parameters of the money supply rule appropriately? (Leaving aside the question whether it *should* do so)

How do we solve this model?

- Use AD and AS to solve for the price level:

$$\alpha_0+\alpha_1(p_t-E_{t-1}p_t)+u_t = \beta_0+\beta_1(m_t-p_t)+\beta_2 E_{t-1}(p_{t+1}-p_t)+v_t$$

- Hence:

$$p_t = \frac{\beta_0 - \alpha_0 + \beta_1 m_t + \alpha_1 E_{t-1} p_t + \beta_2 E_{t-1}\left[p_{t+1} - p_t\right] + v_t - u_t}{\alpha_1 + \beta_1} \tag{S4}$$

How do we solve this model?

- Take expectations based on information set dated $t-1$ (this is not just a lucky guess–observe that we need the price error, $p_t - E_{t-1}p_t$, in the AS curve):

$$E_{t-1}p_t = E_{t-1}\left(\frac{\beta_0 - \alpha_0 + \beta_1 m_t + \alpha_1 E_{t-1}p_t + \beta_2 E_{t-1}\left[p_{t+1} - p_t\right] + v_t - u_t}{\alpha_1 + \beta_1}\right)$$

  - Parameters are known by the agents and can be taken out of the expectations operator
  - $E_{t-1}E_{t-1}p_t = E_{t-1}p_t$ and $E_{t-1}E_{t-1}p_{t+1} = E_{t-1}p_{t+1}$ (the expectation of a constant is that constant itself)
  - $E_{t-1}v_t = 0$ and $E_{t-1}u_t = 0$ by assumption (no autocorrelation in the shocks)

How do we solve this model?

- Imposing all these results we find:

$$E_{t-1}p_t = \frac{\beta_0 - \alpha_0 + \beta_1 E_{t-1}m_t + \alpha_1 E_{t-1}p_t + \beta_2 E_{t-1}\left[p_{t+1} - p_t\right]}{\alpha_1 + \beta_1} \qquad \text{(S5)}$$

- Recall expression (S4) for the actual price level, p_t:

$$p_t = \frac{\beta_0 - \alpha_0 + \beta_1 m_t + \alpha_1 E_{t-1}p_t + \beta_2 E_{t-1}\left[p_{t+1} - p_t\right] + v_t - u_t}{\alpha_1 + \beta_1} \qquad \text{(S4)}$$

How do we solve this model?

- By deducting (S5) from (S4) we find an expression for the price error:

$$p_t - E_{t-1}p_t = \frac{\beta_1}{\alpha_1 + \beta_1}\left[m_t - E_{t-1}m_t\right] + \frac{1}{\alpha_1 + \beta_1}\left[v_t - u_t\right] \tag{S6}$$

The price is higher than rationally expected if:

  - The money supply is higher than was rationally expected ($m_t > E_{t-1}m_t$)
  - The AD shock was higher than was rationally expected ($v_t > E_{t-1}v_t = 0$)
  - The AS shock was lower than was rationally expected ($u_t < E_{t-1}u_t = 0$)

How do we solve this model?

- By using the MSR agents rationally forecast the money supply in period t:

$$\begin{aligned} E_{t-1}m_t &= \mu_0 + \mu_1 E_{t-1}m_{t-1} + \mu_2 E_{t-1}y_{t-1} + E_{t-1}e_t \\ &= \mu_0 + \mu_1 m_{t-1} + \mu_2 y_{t-1} \end{aligned}$$

- Actual money supply is:

$$m_t = \mu_0 + \mu_1 m_{t-1} + \mu_2 y_{t-1} + e_t \qquad \text{(MSR)}$$

- Hence, the "money surprise" is:

$$m_t - E_{t-1}m_t = e_t \qquad \text{(S7)}$$

How do we solve this model?

- By substituting (S6) and (S7) into the AS curve we obtain the REH solution for output:

$$y_t = \alpha_0 + \frac{\alpha_1 \beta_1 e_t + \alpha_1 v_t + \beta_1 u_t}{\alpha_1 + \beta_1}$$

- We have derived a "disturbing result": output does not depend on any of the policy variables (the μ_i coefficients)! **Hence, the policy maker cannot influence output in this model!** This is the strong **policy ineffectiveness proposition** (PIP)
- **Lucas critique**: the macroeconometric models used in the 1960 and 1970s are no good for policy simulation because their coefficients are not invariant with respect to the policy stance. Once you attempt to use the macroeconometric model for setting policy its parameters will change

Should the PIP be taken seriously?

Or: Are macroeconomists useless?

- To disprove a supposedly general proposition all that is needed is one counter-example
- The Keynesian economist Stanley Fischer provided this counter-example
- Key idea: if there are nominal (non-indexed) wage contracts which are renewed less frequently than new information becomes available, the government has an informational advantage over the public
- Result: stabilization is possible (PIP invalid) and is desirable (raises welfare)
- In order to show that the informational advantage of the policy maker is crucial we first study the case with one-period contracts. Then we go on to the general case with two-period contracts

Model 1: Single-period nominal wage contracts

- All variables in logarithms
- The AD curve is monetarist (no Tobin effect and no effect of government consumption):

$$y_t = m_t - p_t + v_t \qquad \text{(AD)}$$

- AD shock is assumed to display autocorrelation:

$$v_t = \rho_V v_{t-1} + \eta_t, \qquad |\rho_V| < 1$$
$$\eta_t \sim N(0, \sigma_\eta^2)$$

Model 1: Single-period nominal wage contracts

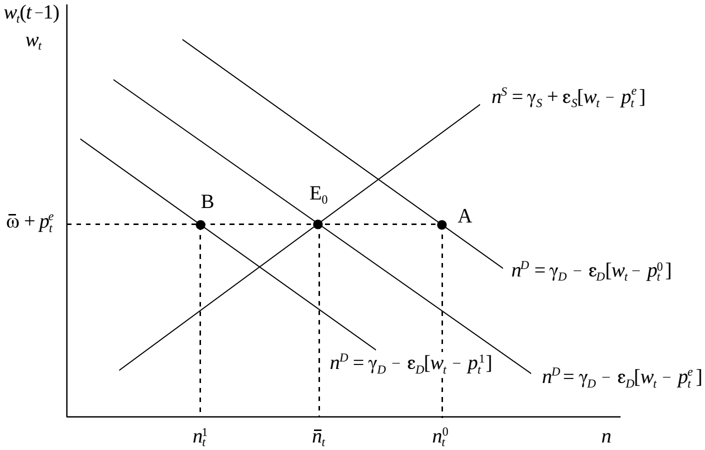
- The nominal wage is set in period $t-1$ to hold for period t is such that full employment of labour is expected in period t
- The equilibrium real wage rate is normalized to unity (so that its logarithm $\bar{\omega}$ is zero):

$$w_t(\underbrace{t-1}_{(a)}) = E_{t-1}p_t \tag{S8}$$

(a) Date of contract settlement

- See **Figure 5.6**

Figure 5.6: Wage setting with single-period contracts



Model 1: Single-period nominal wage contracts

- The supply of output depends on the actual real wage in period t (labour demand determines the quantity of labour traded and thus output)

$$y_t = -\left[w_t(t-1) - p_t\right] + u_t \qquad \text{(S9)}$$

- The shock in output supply is autocorrelated:

$$u_t = \rho_U u_{t-1} + \varepsilon_t, \qquad |\rho_U| < 1$$
$$\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$$

- Inserting (S8) into (S9) yields a kind of Lucas supply curve:

$$y_t = \left[p_t - E_{t-1}p_t\right] + u_t \qquad \text{(LSC)}$$

Model 1: Single-period nominal wage contracts

- The policy rule of the monetary policy maker is given by:

$$m_t = \sum_{i=1}^{\infty} \mu_{1i} u_{t-i} + \sum_{i=1}^{\infty} \mu_{2i} v_{t-i} \qquad \text{(MSR)}$$

  - In *principle* policy maker can react to all past shocks in aggregate demand and supply
  - In *practice* it is only needed to react to shocks lagged once and lagged twice, so that $\mu_{1i} = \mu_{2i} = 0$ for $i = 3, 4, \cdots, \infty$

Model 1: Single-period nominal wage contracts

- Rational expectations solution for the price error is:

$$p_t - E_{t-1}p_t = \tfrac{1}{2}\Big[\underbrace{(m_t - E_{t-1}m_t)}_{(a)} + \underbrace{(v_t - E_{t-1}v_t)}_{(b)} - \underbrace{(u_t - E_{t-1}u_t)}_{(c)}\Big]$$
$$= \tfrac{1}{2}\left[\eta_t - \varepsilon_t\right]$$

(a) $m_t - E_{t-1}m_t = 0$ as the MSR only contains variables that are in the information set of the agent at time $t-1$
(b) $v_t - E_{t-1}v_t = \eta_t$ as agents know the stochastic process for v_t
(c) $u_t - E_{t-1}u_t = \varepsilon_t$ as agents know the stochastic process for u_t

- Rational expectations solution for output is:

$$y_t = \tfrac{1}{2}\left[\eta_t - \varepsilon_t\right] + u_t$$

Model 1: Single-period nominal wage contracts

- Conclusion: for model 1 we still have PIP
- The policy parameters (μ_{1i} and μ_{2i}) do not influence aggregate output at all despite the fact that there are nominal contracts
- The reason is that the policy maker is as much in the dark as the private agents are and thus has no informational advantage

Model 2: Two-period overlapping nominal wage contracts

- AD curve and MSR the same as before:

$$y_t = m_t - p_t + v_t \tag{AD}$$

$$m_t = \sum_{i=1}^{\infty} \mu_{1i} u_{t-i} + \sum_{i=1}^{\infty} \mu_{2i} v_{t-i} \tag{MSR}$$

- Nominal wage contracts
  - Run for two periods
  - Each period, half of the work force is up for renewal of their contract
  - Wage set such that market clearing of labour market is expected

Model 2: Two-period overlapping nominal wage contracts

- Nominal wage contracts (continued).
  - In period t half of the work force receive $w_t(t-1)$ and the other half receives $w_t(t-2)$:

$$w_t(t-1) \equiv E_{t-1}p_t$$
$$w_t(t-2) \equiv E_{t-2}p_t$$

  - Half of the work force is on wages based on "stale information" (i.e. dated $t-2$)
- Firms are perfectly competitive (law of one price). Aggregate supply is:

$$y_t = \frac{1}{2}\underbrace{\left[p_t - w_t(t-1) + u_t\right]}_{(a)} + \frac{1}{2}\underbrace{\left[p_t - w_t(t-2) + u_t\right]}_{(b)} \quad \text{(S10)}$$

(a) Supply by firms which renewed their workers' contract in period $t-1$
(b) Supply by firms which renewed their workers' contract in period $t-2$

Model 2: Two-period overlapping nominal wage contracts

- By substituting $w_t(t-1)$ and $w_t(t-2)$ into (S10) we obtain the AS curve when there are overlapping nominal wage contracts:

$$y_t = \tfrac{1}{2}\left[p_t - E_{t-1}p_t\right] + \tfrac{1}{2}\left[p_t - E_{t-2}p_t\right] + u_t$$

- The rational expectations solution for output is:

$$\begin{aligned} y_t &= \tfrac{1}{2}\left[\eta_t + \varepsilon_t\right] + \rho_U^2 u_{t-2} \\ &\quad + \tfrac{1}{3}\left[\mu_{21} + \rho_V\right]\eta_{t-1} + \tfrac{1}{3}\left[\mu_{11} + 2\rho_U\right]\varepsilon_{t-1} \end{aligned}$$

  - First line contains no policy parameters. This is unavoidable turbulence in the economy
  - Second line contains policy parameters (μ_{21} and μ_{11}). The policy maker can offset the effects of η_{t-1} and ε_{t-1} by choosing $\mu_{21} = -\rho_V$ and $\mu_{11} = -2\rho_U$
  - PIP is refuted by this example as output can be stabilized

Model 2: Two-period overlapping nominal wage contracts

- Stabilization is not only feasible, it is highly desirable as it improves economic welfare (as proxied by the asymptotic variance of output):

$$\sigma_Y^2 \equiv \sigma_\varepsilon^2 \left[\frac{1}{4} + \frac{\rho_U^4}{1-\rho_U^2} + \frac{1}{9} \Big(\underbrace{\mu_{11} + 2\rho_U}_{(a)} \Big)^2 \right] + \sigma_\eta^2 \left[\frac{1}{4} + \frac{1}{9} \Big(\underbrace{\mu_{21} + \rho_V}_{(b)} \Big)^2 \right]$$

(a) By setting $\mu_{11} = -2\rho_U$ this term can be eliminated. Intuition: if $\varepsilon_{t-1} > 0$ (positive innovation to the supply shock process) then the money supply should be reduced somewhat to avoid "overheating" of the economy (counter-cyclical monetary policy)

Model 2: Two-period overlapping nominal wage contracts

- Solution features (continued)

$$\sigma_Y^2 \equiv \sigma_\varepsilon^2 \left[\frac{1}{4} + \frac{\rho_U^4}{1-\rho_U^2} + \frac{1}{9} \Big(\underbrace{\mu_{11} + 2\rho_U}_{(a)} \Big)^2 \right] + \sigma_\eta^2 \left[\frac{1}{4} + \frac{1}{9} \Big(\underbrace{\mu_{21} + \rho_V}_{(b)} \Big)^2 \right]$$

  (b) Similarly, by setting $\mu_{21} = -\rho_V$ this term can be eliminated. Intuition: if $\eta_{t-1} > 0$ (positive innovation to the demand shock process) then the money supply should be reduced somewhat to avoid "overheating" of the economy (counter-cyclical monetary policy)

- The government can improve matters (relative to non-intervention) because it has an informational advantage relative to the public

Test your understanding

****** Self Test ******

Make sure you understand how we obtain the rational expectations solution for output and how we derive the expression for the asymptotic variance of output.

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Punchlines

- REH does not in and of itself imply PIP.
- REH + Classical model $\Rightarrow$ Classical conclusions.
- REH + Keynesian model $\Rightarrow$ Keynesian conclusions.
- REH accepted by virtually all economists (extension of equilibrium idea to expectations).
- .... but we hope for more!