Dynamics in AD-AS

Macroeconomic Theory

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Outline

Aims of this chapter

The principal aim of this chapter is to study the "intrinsic dynamics" in IS-LM type models. Particularly, we look at the following examples:

- The Adaptive Expectations Hypothesis (AEH) and stability in the AD-AS model
- Investment theory and the interaction between the *stock* of capital (K) and the *flow* of investment (I). This is yet another important building block for the course
- The government budget restriction, stability, stock-flow interaction, and multipliers under different financing methods
- Hysteresis and path dependency

What do we mean by stability?

Loose definition: System returns to equilibrium following an exogenous shock

Question: Why are we so interested in stable models?

- Unstable models are rather useless
- The Samuelsonian "correspondence principle" is very handy
- "Backward looking" stability arises naturally in IS-LM type models and is easy to handle
- "Forward looking" stability is a more recently developed form of stability but it can also be handled relatively easily

The AEH and stability in the AS-AD model

Assume that we have a simple continuous-time model in the tradition of the Neo-Keynesian Synthesis:

$$Y = AD(\underset{+}{G}, \underset{+}{M/P}), \qquad AD_G > 0, \quad AD_{M/P} > 0$$

$$Y = Y^* + \phi\left[P - P^e\right], \quad \phi > 0$$

$$\dot{P}^e = \lambda\left[P - P^e\right], \qquad \lambda > 0$$

where $\dot{P}^e \equiv dP^e/dt$ and Y^* is full employment output (output level consistent with full employment in the labour market)

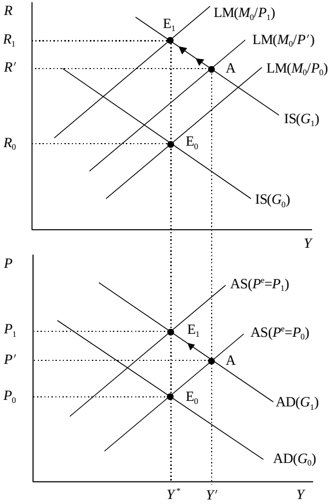
- The AD curve depends positively on both government consumption (G) and on the level of real money balances (M/P)
- The AS curve is upward sloping in the short run because of expectational errors
- Expected price level adapts gradually to expectational error

Graphical stability analysis

Example of graphical stability analysis: Trace the dynamic effects of a permanent increase in government consumption

- See **Figure 3.1** for the graphical derivation. Key effects:
  - $G\uparrow$ so that IS and AD both shift up
  - P^e is given so that short-run equilibrium is at point A
  - In point A, $P^e \neq P$ (expectational disequilibrium)
  - Since $P > P^e$, $\dot{P}^e > 0$ and AS_{SR} starts to shift up
  - Economy moves gradually along the AD curve from A to E_1
- We can conclude from the graph that the model is stable!

Figure 3.1: Fiscal policy under adaptive expectations



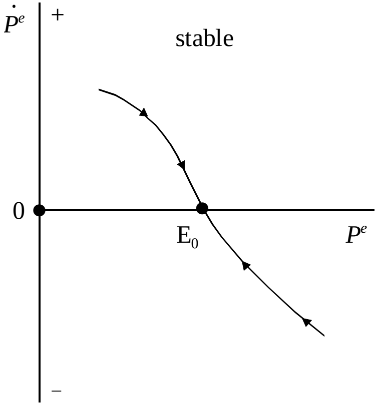
Can we do this analytically?

- This is useful if the model is too complicated to analyze graphically
- Stability holds in our model provided $\dot{P}^e$ dies out (goes to zero)
- In a *phase diagram* the stable and unstable cases look like in **Figure A**
- From the diagram we conclude that we must show that for a stable model the phase diagram slopes downward:

$$\frac{\partial \dot{P}^e}{\partial P^e} < 0 \qquad \text{(stability condition)}$$

- Note that a model may be non-linear. All we do is prove *local stability*, i.e. stability close to an equilibrium.

Figure A: Phase diagram



Can we do this analytically?

- In our model we must take into account that P depends on P^e (and the other exogenous variables):

$$P = \Phi(G, M, Y^*, P^e) \quad \text{(S1)}$$

- We use AD and AS to find $\Phi_{P^e} \equiv \partial P/\partial P^e$ with our implicit function trick:

$$dY = AD_G dG + AD_{M/P}(M/P)\left[\frac{dM}{M} - \frac{dP}{P}\right]$$

$$dY = \phi\left[dP - dP^e\right] + dY^*$$

and solve for dP:

$$dP = \frac{\phi dP^e + AD_G dG + AD_{M/P}(1/P)dM - dY^*}{\phi + AD_{M/P}(M/P^2)}$$

- We conclude that $\partial P/\partial P^e = \phi/\left[\phi + (M/P^2)AD_{M/P}\right]$ which is between 0 and 1

Can we do this analytically?

- The AEH implies:

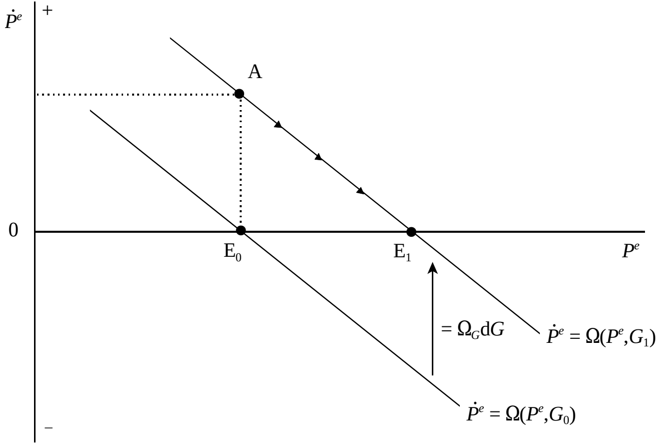
$$\dot{P}^e = \lambda \left[\Phi(\underset{+}{G}, \underset{+}{M}, \underset{-}{Y^*}, \underset{+}{P^e}) - P^e \right] \equiv \Omega(P^e, G, M, Y^*) \quad \text{(S2)}$$

- By partially differentiating (S2) with respect to P^e we find:

$$\begin{aligned} \frac{\partial \dot{P}^e}{\partial P^e} &= \lambda \left[\Phi_{P^e}(G, M, Y^*, P^e) - 1 \right] \\ &= \lambda \left[\frac{\phi}{\phi + (M/P^2) AD_{M/P}} - 1 \right] \\ &= -\lambda \left[\frac{(M/P^2) AD_{M/P}}{\phi + (M/P^2) AD_{M/P}} \right] < 0 \quad \text{(S3)} \end{aligned}$$

- We conclude that $\partial \dot{P}^e / \partial P^e < 0$ so that the model is stable
- We can integrate the stability analysis with the fiscal policy shock in **Figure 3.2**

Figure 3.2: Stability and adaptive expectations



Test your understanding

****** Self Test ******

Phase diagrams are very important in modern macroeconomics. Make absolutely sure you feel confident working with them! If you don't understand these simple (one-dimensional) phase diagrams you will have trouble later on!

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Building block: A first look at investment theory

(Recall our earlier building blocks: demand for labour by firms, supply of labour by households, demand for money by households.) We are now going to start the development of a theory of investment, i.e. the accumulation of *capital goods* (such as machines, PCs, buildings, etcetera) by firms. Basic ingredients:

- Adjustment cost model
- Firms now choose both employment (as in Chapter 1) *and* investment
- Simplifying assumptions: static expectations, perfect competition

Building block: A first look at investment theory

- Production function still given by:

$$Y_t = F(N_t, K_t)$$

  - Need time subscript because investment decision is dynamic
  - Choices made now affect outcomes in the future
  - Example: just like the decision to educate oneself
- Timing: K_t is the capital *stock* at the beginning of period t
- Properties as before: positive but diminishing marginal products ($F_N > 0$, $F_K > 0$, $F_{NN} < 0$, and $F_{KK} < 0$), cooperative factors ($F_{NK} > 0$), and CRTS
- Accumulation identity:

$$\underbrace{K_{t+1} - K_t}_{1} = \underbrace{I_t}_{2} - \underbrace{\delta K_t}_{3}$$

Net investment (term **1**) equals gross investment (term **2**) minus depreciation of existing capital (term **3**)

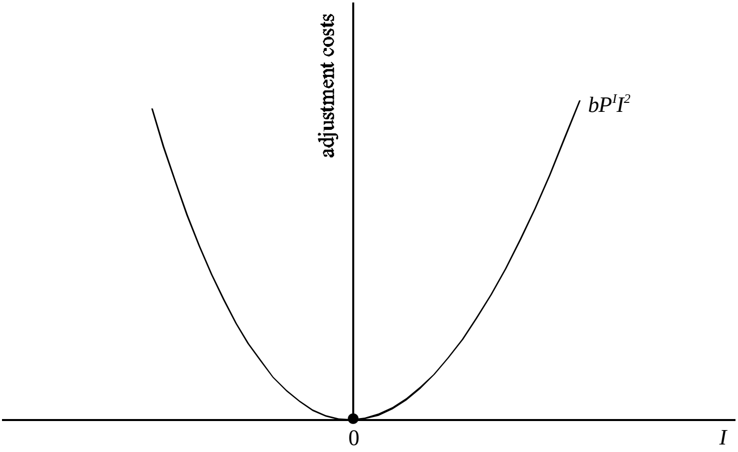
Objective of the Firm

- The representative firm's manager maximizes the present value of net payments to owners of the firm ("share holders") using the market rate of interest to discount future payments (Modigliani-Miller Theorem)
- Profit in period t is:

$$\Pi_t = \underbrace{PF(N_t, K_t)}_{1} - \underbrace{WN_t}_{2} - \underbrace{P^I I_t}_{3} - \underbrace{bP^I I_t^2}_{4}$$

Profit (or cash flow) equals revenue (term **1**) minus the wage bill (term **2**) minus the purchase cost of new capital (term **3**) minus the quadratic adjustment costs (term **4**). See **Figure 3.3**

Figure 3.3: Adjustment costs of investment



Share Value Maximization

- Let us call the planning period "today" and normalize it to $t = 0$
- The value of the firm in the stock market is:

$$\bar{V}_0 \equiv \sum_{t=0}^{\infty} \left(\frac{1}{1+R} \right)^t \Pi_t$$
$$= \sum_{t=0}^{\infty} \left(\frac{1}{1+R} \right)^t \left[PF(N_t, K_t) - WN_t - P^I I_t - bP^I I_t^2 \right]$$

- The firm must choose paths for N_t and K_t (and thus for Y_t) such that $\bar{V}_0$ is maximized subject to the accumulation identity (and the initial capital stock, K_0)

Share Value Maximization

- To solve the problem we use the method of Lagrange multipliers. The Lagrangian is:

$$\mathcal{L}_0 \equiv \sum_{t=0}^{\infty} \left(\frac{1}{1+R}\right)^t \left[PF(N_t, K_t) - WN_t - P^I I_t - bP^I I_t^2\right] - \sum_{t=0}^{\infty} \frac{\lambda_t}{(1+R)^t} \left[K_{t+1} - (1-\delta)K_t - I_t\right]$$

  - We need a whole *path* of Lagrange multipliers – λ_t is the one relevant for the constraint in period t
  - Note that we scale the Lagrange multipliers in order to facilitate interpretation later on

First-Order Conditions

- First-order necessary conditions (FONCs) (for $t = 0, 1, 2, ...$):

$$\frac{\partial \mathcal{L}_0}{\partial N_t} = \left(\frac{1}{1+R}\right)^t [PF_N(N_t, K_t) - W] = 0$$

$$\frac{\partial \mathcal{L}_0}{\partial K_{t+1}} = \left(\frac{1}{1+R}\right)^t \left[\frac{PF_K(N_{t+1}, K_{t+1}) + \lambda_{t+1}(1-\delta)}{1+R} - \lambda_t\right] = 0$$

$$\frac{\partial \mathcal{L}_0}{\partial I_t} = \left(\frac{1}{1+R}\right)^t \left[-P^I - 2bP^I I_t + \lambda_t\right] = 0$$

  - Note the timing in the expression for $\partial \mathcal{L}_0 / \partial K_{t+1}$!

Interpretation FONCs

- There are no adjustment costs on labour. Hence the firm can vary employment freely in each period such that:

$$PF_N(N_t, K_t) = W$$

- The FONC for investment yields (for adjacent periods t and $t+1$):

$$\lambda_t = P^I \cdot [1 + 2bI_t]$$
$$\lambda_{t+1} = P^I \cdot [1 + 2bI_{t+1}]$$

- The FONC for capital is:

$$PF_K(N_{t+1}, K_{t+1}) + \lambda_{t+1}(1 - \delta) - \lambda_t(1 + R) = 0$$

Interpretation FONCs

- Substituting λ_t and λ_{t+1} into the FONC for capital gives:

$$\begin{aligned} 0 = & PF_K(N_{t+1}, K_{t+1}) + P^I \cdot [1 + 2bI_t](1-\delta) \\ & - P^I \cdot [1 + 2bI_t](1+R) \qquad \Leftrightarrow \end{aligned}$$

$$I_{t+1} = \frac{1+R}{1-\delta} I_t - \frac{PF_K(N_{t+1}, K_{t+1}) - P^I(R+\delta)}{2bP^I(1-\delta)} \qquad \text{(S4)}$$

- Eq. (S4) is an *unstable* difference equation: the coefficient for I_t is greater than 1 (as $R > 0$ and $0 < \delta < 1$)
- In general $I_t \to +\infty$ or $I_t \to -\infty$. But these are economically non-sensical solutions because adjustment costs for the firm will explode and thus firm profits and the value of the firm will go to $-\infty$

Interpretation FONCs

- But (S4) pins down only one economically sensible investment policy, namely the constant policy, for which $I_{t+1} = I_t = I$
- Solving (S4) for this policy yields:

$$I = \frac{1}{2b}\left[\frac{PF_K(N,K)}{P^I(R+\delta)} - 1\right] \tag{S5}$$

where we have dropped the time subscripts to indicate that (S5) is a steady-state investment policy (we analyze the non-steady-state case in Chapter 4)

Interpretation FONCs

- Let us assume that $P^I = P$ (single good economy; no investment subsidy). Then (S5) simplifies to:

$$I = \frac{1}{2b}\left[\frac{F_K(N,K)}{R+\delta} - 1\right]$$

  - If there are no adjustment costs ($b \to 0$) then the firm expresses a demand for *capital*. The demand for investment is not well-defined in that case, because there is no punishment for the firm in adjusting its stock of capital freely (i.e. $I_t \to +\infty$ or $I_t \to -\infty$ are no longer disastrous in that case)
  - Formally, if $b \to 0$ then so must the term in square brackets:

$$\frac{F_K(N,K)}{R+\delta} - 1 = 0 \qquad \Leftrightarrow \qquad F_K = R + \delta$$

  - Notice the parallel with the expression for labour demand in this case (the firm rents the use of the capital goods)

Summary Investment Model

- With adjustment costs, however, we have a well-defined investment equation which we write generally as:

$$I = I(\underset{-}{R}, \underset{-}{K}, \underset{+}{Y}), \quad I_R < 0,\ I_K < 0,\ I_Y > 0$$

  - *Example #1*: Cobb-Douglas production function.
    - $Y = N^{\alpha} K^{1-\alpha}$ (with $0 < \alpha < 1$)
    - $F_K = (1-\alpha) Y/K$
  - *Example #2*: CES production function.
    - $Y \equiv \left[(1-\alpha) K^{(\sigma-1)/\sigma} + \alpha N(t)^{(\sigma-1)/\sigma} \right]^{\sigma/(\sigma-1)}$ with $\sigma \geq 0$
    - $F_K = (1-\alpha) (Y/K)^{\sigma}$

Augmented IS-LM Model

- We can study the stock-flow interaction on the demand side of the economy, in the IS-LM model
- The model is:

$$\begin{aligned} Y &= C(Y - T(Y)) + I(R, K, Y) + G \\ M/P &= l(Y, R) \\ \dot{K} &= I(R, K, Y) - \delta K \end{aligned}$$

- We keep P and M fixed throughout
- IS-LM equilibrium yields:

$$\begin{aligned} Y &= \Phi(\underset{-}{K}, \underset{+}{G}) \\ R &= \Psi(\underset{-}{K}, \underset{+}{G}) \end{aligned}$$

Test your understanding

****** Self Test ******

Draw IS-LM diagrams to rationalize the partial derivative effects for Y and R. Use **Figure 3.4** *to do so.*

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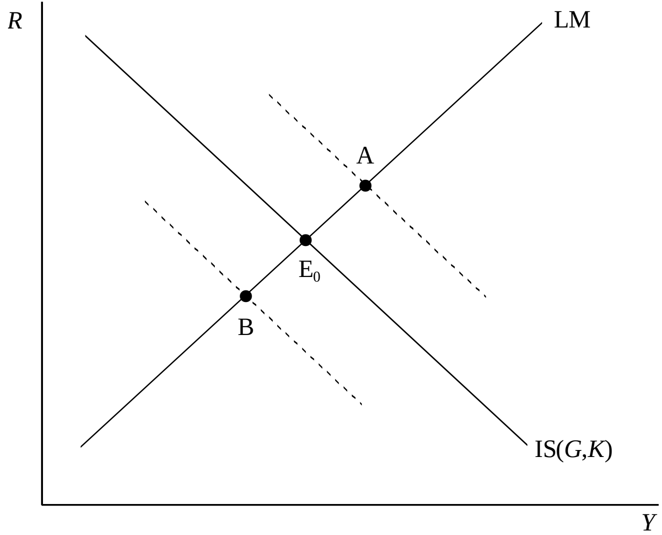
Figure 3.4: Comparative static effects in the IS-LM model



Capital Accumulation and Stability

- Capital dynamics is governed by:

$$\dot{K} = I\big(\underbrace{\Psi(P,K,G,M)}_{R}, K, \underbrace{\Phi(P,K,G,M)}_{Y}\big) - \delta K$$
$$\equiv \Omega(K,G)$$

- Note that the capital stock, K, appears in no less than *four* places on the right-hand side
- Hence, checking stability (by computing $\partial \dot{K}/\partial K$ and proving it is negative) is much more difficult
- A graphical approach will not help!

Capital Accumulation and Stability

- Formally we find:

$$d\dot{K} = \Omega_K dK + \Omega_G dG \tag{S6}$$

with the partial derivatives:

$$\Omega_K \equiv \underset{-}{I_R}\underset{-}{\Psi_K} + \underset{-}{I_K} + \underset{+}{I_Y}\underset{-}{\Phi_K} - \underset{+}{\delta} \tag{S7}$$

$$\Omega_G \equiv I_R \Psi_G + I_Y \Phi_G \tag{S8}$$

- Not at all guaranteed that Ω_K is negative (as is required for stability); the term $I_R \Psi_K > 0$ which is a “destabilizing” influence
- Appeal to the **Samuelsonian Correspondence Principle** (believe and use only stable models) and simply assume that $\Omega_K \equiv \partial \dot{K}/\partial K < 0$. This gets you information that is useful to determine the long-run effect of fiscal policy.

Stable Adjustment to Fiscal Policy Shock

- From (S6) we find that, assuming stability, $d\dot{K} = 0$ in the long run so that the long-run effect on capital is:

$$\left(\frac{dK}{dG}\right)^{LR} = -\frac{\Omega_G}{\Omega_K} = \frac{\overset{-}{I_R}\overset{+}{\Psi_G} + \overset{+}{I_Y}\overset{+}{\Phi_G}}{-\Omega_K}$$

  where the denominator is positive for the stable case (since $\Omega_K < 0$)
- The long-run effect on capital of an increase in government consumption is ambiguous

Stable Adjustment to Fiscal Policy Shock

- Heated debate in the 1970s between monetarists (like Friedman) and Keynesians (like Tobin) (a.k.a. the "battle of the slopes"):
  - *Friedman*: a strong interest rate effect on investment ($|I_R|$ large), and a large effect on the interest rate but a small effect on output of a rise in government spending (Ψ_G large and Φ_G small). Consequently, a monetarist might suggest that $\partial \dot{K}/\partial G$ is negative: **crowding out**
  - *Tobin*: $|I_R|$ small, Ψ_G small, and Φ_G large, so that $\partial \dot{K}/\partial G > 0$: **crowding in**
  - Correspondence Principle does not settle the issue. Econometric studies could do so.
- In **Figures 3.5 and 3.6** illustrate the two cases

Figure 3.5: The effect on capital of a rise in public spending

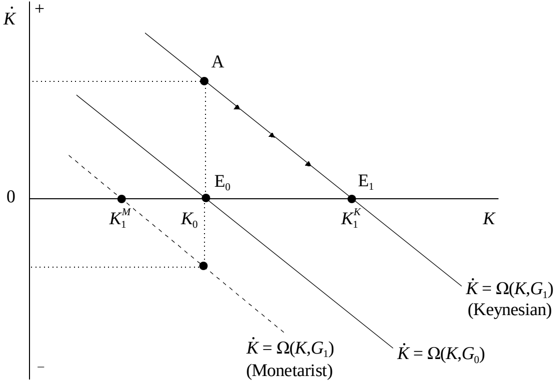
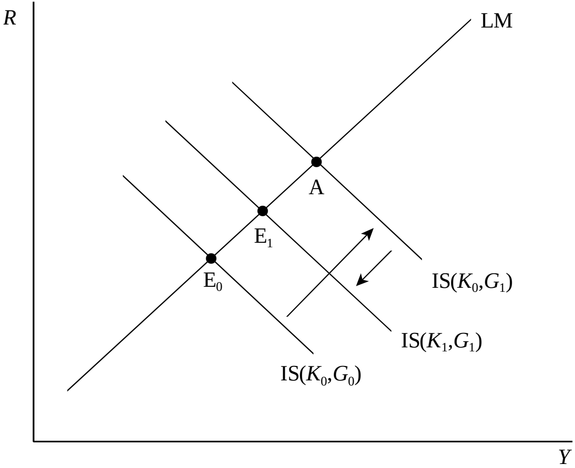


Figure 3.6: Capital accumulation and the Keynesian effects of fiscal policy



Intrinsic Dynamics and the Government Budget Constraint

- IS-LM is a little strange because:
  - It combines flow concepts (IS) and stock concepts (LM) in one diagram
  - It cannot be used to study effect of government financing method
- Blinder and Solow (1973) show how the IS-LM model can be extended with a government budget restriction. With their model we can study:
  - Money creation
  - Tax financing
  - Bond financing

Key ingredients of the Blinder-Solow model

- Fixed price level, $P = 1$ (horizontal AS curve)
- Special type of bond, the consol, pays 1 euro from now until perpetuity
- If the interest rate is R the price of the bond would be:

$$P_B = \int_0^\infty 1e^{-R\tau} d\tau = -(1/R)\left[e^{-R\tau}\right]_0^\infty = \frac{1}{R}$$

- If there are B consols in existence than the "coupon payments" at each instant is $B \times 1$ euros
- If the government emits new consols, $\dot{B} > 0$, then it receives $\dot{B} \times P_B$ in revenue from the bond sale
- If the government issues new money, then $\dot{M} > 0$

Key ingredients of the Blinder-Solow model

- The government budget constraint is:

$$G + B = T + \dot{M} + \frac{1}{R} \cdot \dot{B}$$

  Government consumption plus coupon payments equals tax revenue plus money issuance plus revenue from new bond sales.

- Other changes to the IS-LM model:

$$\begin{aligned} T &= T(Y+B), & 0 < T_{Y+B} < 1 \\ A &\equiv \bar{K} + M/P + B/R, & \\ C &= C(Y+B-T, A), & 0 < C_{Y+B-T} < 1,\ C_A > 0 \\ M/P &= l(Y, R, A), & l_Y > 0,\ l_R < 0,\ 0 < l_A < 1 \end{aligned}$$

Key ingredients of the Blinder-Solow model

- New IS curve:

$$Y = C\big[\underbrace{Y + B - T(Y+B)}_{1}, \underbrace{\bar{K} + M/P + B/R}_{2}\big] + I(R) + G$$

where term **1** is household disposable income, and term **2** is total wealth

- We keep $\bar{K}$ fixed
- "Quasi-reduced form" expressions for Y and R can be derived in the usual way:

$$Y = \Phi(\underset{+}{G}, \underset{?}{B}, \underset{+}{M})$$

$$R = \Psi(\underset{+}{G}, \underset{+}{B}, \underset{?}{M})$$

Key ingredients of the Blinder-Solow model

- We consider two prototypical cases
- Pure money financing ($\dot{M} \neq 0$ and $\dot{B} = 0$)
- Pure bond financing ($\dot{M} = 0$ and $\dot{B} \neq 0$)
- Key issues:
  - Is the model stable?
  - Relation between financing method and the government spending multiplier
  - How do the two cases compare?

Pure money financing ($\dot{M} \neq 0$, $\dot{B} = 0$)

- Money financing is stable:

$$\frac{\partial \dot{M}}{\partial M} \equiv -T_{Y+B}\Phi_M < 0$$

- Boost in government consumption causes an initial government deficit:

$$\frac{\partial \dot{M}}{\partial G} \equiv (1 - T_{Y+B}\Phi_G) > 0$$

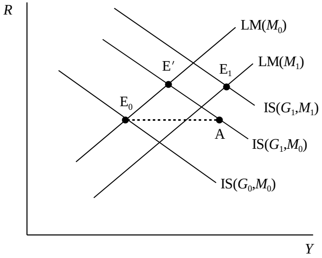
- Long-run multiplier exceeds short-run multiplier:

$$\left(\frac{dY}{dG}\right)_{MF}^{LR} \equiv \frac{1}{T_{Y+B}} > \Phi_G \equiv \left(\frac{dY}{dG}\right)_{MF}^{SR}$$

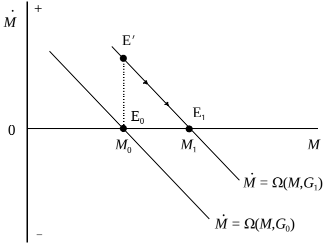
- Economic intuition: both IS and LM shift out, $Y\uparrow$, $T(Y)\uparrow$, deficit closes and $\dot{M} = 0$
- See **Figure 3.7** for the graphical illustration

Figure 3.7: The effects of fiscal policy under money finance

(a) IS-LM diagram



(b) Phase diagram



Pure bond financing ($\dot{M} = 0$, $\dot{B} \neq 0$)

- Bond financing may be unstable:

$$\frac{\partial \dot{B}}{\partial B} = 1 - \underset{0<\cdot<1}{T_{Y+B}}(1 + \underset{?}{\Phi_B}) \gtreqless 0$$

Economic intuition: Φ_B is ambiguous because $B \uparrow$ shifts IS to the right (via consumption) but LM to the left (via money demand). Net effect ambiguous.

- But Samuelsonian "correspondence principle" helps:

$$\begin{aligned} \frac{\partial \dot{B}}{\partial B} &< 0 \\ \Leftrightarrow \quad & 1 - T_{Y+B}(1 + \Phi_B) < 0 \\ \Leftrightarrow \quad & \Phi_B > \frac{1 - T_{Y+B}}{T_{Y+B}} > 0 \end{aligned}$$

Pure bond financing

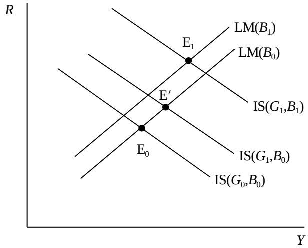
- For the stable case the long-run multiplier again exceeds the short-run multiplier:

$$\underbrace{\left(\frac{dY}{dG}\right)^{LR}_{BF}}_{\Phi_G + \Phi_B\left(\frac{1 - T_{Y+B}\Phi_G}{1 - T_{Y+B}(1+\Phi_B)}\right)} > \underbrace{\left(\frac{dY}{dG}\right)^{SR}_{BF}}_{\Phi_G}$$

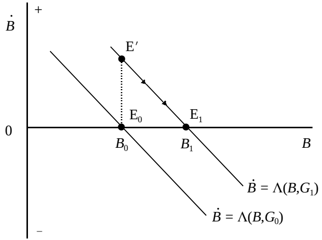
- See **Figure 3.8** for the graphical illustration

Figure 3.8: The effects of fiscal policy under (stable) bond financing

(a) IS-LM diagram



(b) Phase diagram



Comparison money financing and bond financing

- The long-run (stable) bond-financed multiplier exceeds the long-run money-finance multiplier:

$$\underbrace{\left(\frac{dY}{dG}\right)_{BF}^{LR}}_{\Phi_G - \Phi_B\left(\frac{1 - T_{Y+B}\Phi_G}{1 - T_{Y+B}(1+\Phi_B)}\right)} > \underbrace{\left(\frac{dY}{dG}\right)_{MF}^{LR}}_{\frac{1}{T_{Y+B}}}$$

- Economic intuition: under bond financing both increase in G and the additional interest payments (increase in B) must eventually be covered by higher tax receipts
- Since $T = T(Y)$, it must be the case that Y rises by more
- See **Figure 3.9** for the graphical illustration

Figure 3.9: Long run effects of fiscal policy under different financing modes