Chapter 6

The Sources of Growth and the Solow Model
Preview

• To examine the Solow growth model as the basis of modern research on economic growth

• To understand the importance of saving, population growth and productivity growth in the Solow Model

• To understand the sources of growth with growth accounting
Economic Growth Around the World

• Since 1960, the United States has remained the richest country with steady economic growth
• Convergence occurred when different with different initial levels of per capita income gravitated to a similar income level
• Some “growth miracle” countries in Asia have shown “convergence big time”
• Some African countries (Kenya, Nigeria) have experienced a “growth disaster”
FIGURE 6.1  Real Per Capita GDP in Ten Countries

Note: The vertical axis in this figure is a ratio scale in which equal distances reflect the same percentage change, so that the slope of a curve indicates how fast an economy is growing.

Source: Penn World Tables in Federal Reserve Bank of St. Louis, FRED Database.
http://research.stlouisfed.org/fred2/
The Solow Growth Model

- Robert Solow developed the **Solow growth model** in the 1950s to explain how saving rates and population growth determine capital accumulation, which in turn affects economic growth
Building Blocks of the Solow Growth Model

- The Cobb-Douglas aggregate production function with constant returns scale (recall Chapter 3):

\[ Y_t = F(K_t, L_t) = AK_t^{0.3}L_t^{0.7} \]

where

- \( Y_t \) = output at time \( t \)
- \( K_t \) = capital stock at time \( t \)
- \( L_t \) = labor at time \( t \)
- \( A \) = available technology, total factor productivity
Box: Time Subscripts

- The time subscript \((t)\) allows us to be more specific about the timing of a variable
- The subscript \(t, t-1, \) or \(t+1\) tells us in which period something is happening
Building Blocks of the Solow Growth Model (cont’d)

• Given a fixed level of labor \((L)\), the Solow model can be expressed in per-worker terms:

\[
y_t = \frac{Y_t}{L_t} \quad \text{(output per worker)}
\]

\[
c_t = \frac{C_t}{L_t} \quad \text{(consumption per worker)}
\]

\[
i_t = \frac{I_t}{L_t} \quad \text{(investment per worker)}
\]

\[
k_t = \frac{K_t}{L_t} \quad \text{(capital per worker or capital-labor ratio)}
\]
Building Blocks of the Solow Growth Model (cont’d)

- Production function in per-worker form:

\[
y_t = \frac{Y_t}{L_t} = \frac{AK_t^{0.3} L_t^{0.7}}{L_t} = \frac{AK_t^{0.3}}{L_t^{0.3}} = A k_t^{0.3}
\]
FIGURE 6.2 Per-Worker Production Function

The slope falls as the capital-labor ratio rises.

\[ y_t = Ak_t^{0.3} \]
Building Blocks of the Solow Growth Model (cont’d)

• Assuming a closed economy and that government spending is zero, the total demand for output is:

\[ y_t = c_t + i_t \]

• Further assume the saving rate \((s)\) is a fixed fraction of income, so that saving is:

\[ y_t - c_t = sy_t \]
Building Blocks of the Solow Growth Model (cont’d)

• The output demand equation means \( i_t = y_t + c_t \)
  so that:

\[
i_t = sy_t
\]

• Substituting the per worker production function into the above equation gives an investment function, which relates per capita investment to per capita capital stock:

\[
i_t = sA k_t^{0.3}
\]
FIGURE 6.3 The Solow Diagram

Capital per worker increases to reach the steady-state levels of $k^*$ and $i^*$. 

Capital per worker decreases to reach the steady-state levels of $k^*$ and $i^*$. 

Investment and Depreciation 

Depreciation, $\delta k_t$ 

Investment $= sA k_t^{0.3}$ 

$\delta k^*$ 

$\delta k^*$ 

$\delta k^*$ 

$\delta k^*$ 

Capital-Labor Ratio, $k_t$ 

$k_1$ 

$k^*$ 

$k_2$ 

$i_1$ 

$i_2$
Building Blocks of the Solow Growth Model (cont’d)

- The capital stock is determined by: \( i_t = y_t + c_t \)
  - Investment
  - Depreciation

- The Solow model assumes the **depreciation rate** as a constant fraction \( \delta \) of capital

- **Capital-accumulation equation** shows the change in the capital stock per worker:

  \[
  \Delta k_t = i_t - \delta k_t
  \]
  
  Change in capital stock per worker = Investment per worker – Depreciation per worker
• Substituting in for investment from the investment function, the capital-accumulation equation becomes:

$$\Delta k_t = sA k_t^{0.3} - \delta k_t$$

• The **steady state** occurs when $\Delta k_t = 0$ so that:

$$sA k_t^{0.3} = \delta k_t$$

Investment = Depreciation
Dynamics of the Solow Model

- The steady-state level of capital per worker is $k^*$ in the **Solow diagram** (Figure 6.3)
- If an economy’s initial capital-labor ratio is at $k_1$, then $\Delta k_t > 0$ so that $k_t$ rises over time until $k_t = k^*$
- If an economy’s initial capital-labor ratio is at $k_2$, then $\Delta k_t < 0$ so that $k_t$ falls over time until $k_t = k^*$
- Output per worker also moves its steady-state value $y^*$ (point S in Figure 6.4) as $k_t$ moves toward its steady-state value
FIGURE 6.4 Output and Consumption in the Solow Model

Output per worker, Investment and Depreciation

\[ y_2 = Ak_2^{0.3} \]
\[ y^* = Ak^{*0.3} \]
\[ y_1 = Ak_1^{0.3} \]
\[ i^* = \delta k^* \]

Consumption:
\[ c^* = (1 - s)y^* \]

Depreciation:
\[ \delta k_t \]

Investment:
\[ sAk_t^{0.3} \]

Capital per worker and output decrease over time to reach the steady-state levels of \( k^* \) and \( y^* \).
Because $c_t = (1-s)y_t$, consumption per worker also reaches its steady state $c_t = c^*$ when $y_t = y^*$

The steady state at $k^*$, $c^*$, and $y^*$ is where the economy will move to and stay if it initially starts away from the steady state at $k_t = k^*$

In other words, the steady state is where the economy converges to in the long run and so is the long-run equilibrium for the economy.
FIGURE 6.5 “Bathtub Model” of the Steady State

Investment, $sAK_t^{0.3}$
Capital = $k_t$
Depreciation = $\delta k_t$
Convergence in the Solow Model

• The Solow model suggests that similar economies will experience convergence

  – Countries with low initial levels of capital and output per worker will grow rapidly as $k_t$ and $y_t$ will rise until they reach their steady state values
  
  – Countries with high initial levels of capital and output per worker will grow slowly as $k_t$ and $y_t$ will fall until they reach their steady state values
Application: Evidence on Convergence, 1960-2012

• There is strong evidence of convergence among wealthy nations.

• Evidence of convergence breaks down when we extend the analysis to a larger group of rich and poor countries, perhaps because many of those economies are not similar mostly due to differences in productivity.
FIGURE 6.6 Evidence on Convergence, 1960-2012 (a)

Panel (a) Convergence Among OECD Countries

FIGURE 6.6 Evidence on Convergence, 1960-2012 (b)

Box: War, Destruction, and Growth Miracles

- Immediately after World War II, the levels of capital per worker in Germany and Japan were very low.
- The Solow growth model correctly predicts that the levels of capital per worker in both countries would rise very rapidly, boosting their income per worker through the early 1970s.
- After 1972, the growth rates of both economies slowed down as their per-capital income became close to that of other rich countries.
Saving Rate Changes in the Solow Model

• Suppose $s$ increases when the economy is at its steady state:
  – *The increase in saving results in higher steady-state levels of capital and output per worker*
  – *Changes in the saving rate affect the level of capital and output per worker, but not the long-run growth rate of these variables*
• Assuming that the ratio of workers to the population is similar across countries, then:
  – the higher is a country’s national saving rate and hence the higher is its level of investment relative to income, the higher is its per capita income
FIGURE 6.7 Response to an Increase in the Saving Rate (a)

Step 1. An increase in the saving rate raises the investment function...

Step 2. raising capital per worker to a steady-state level $k_2^*$. 

Panel (a) Solow Diagram

Investment and Depreciation

$\delta k_1$

$s_2 Ak_1^{0.3}$

$s_1 Ak_1^{0.3}$

$k_1^*$

$k_2^*$
FIGURE 6.7 Response to an Increase in the Saving Rate (b)

Panel (b) Movement of the Capital-Labor Ratio and Output Per Worker Over Time

Step 1. An increase in the saving rate causes capital and output per worker to rise...

Step 2. until capital per worker reaches its steady state level of $k^*_2$, where both capital and output per worker stop rising.
FIGURE 6.8 International Evidence on the Relationship of Per Capita Income and the Saving Rate

Population Growth in the Solow Model

• Suppose population growth and thus the size of the labor force increases at a rate of \( n \) over time when the capital stock is kept constant.

• As the growth in labor force leads to less capital per worker, capital dilution \((nk_t)\) occurs in the capital-accumulation equation:

\[
\Delta k_t = sA k_t^{0.3} - \delta k_t - nk_t
\]

\[
= sA k_t^{0.3} - (\delta + n)k_t
\]
Population Growth in the Solow Model (cont’d)

• Given the population growth rate at \( n \), the depreciation term in the steady state equation is now \( d n \)

• In the Solow diagram, the steady-state level of the capital-labor ratio \( k^* \) is now at the intersection of the investment function and the \( (d+n)k_t \) line
  - If \( k_t < k^* \), then investment is greater than \( (d+n)k_t \) and so \( k_t \) rises
  - If \( k_t > k^* \), then investment is less than \( (d+n)k_t \) and so \( k_t \) falls
FIGURE 6.9 Solow Diagram with Population Growth

Investment and Depreciation plus Capital Dilution

\[ i^* = (\delta + n)k^* \]

Depreciation plus Capital Dilution, \((\delta + n)k_i\)

Capital per worker increases to reach the steady-state levels of \(k^*\) and \(i^*\).

Capital per worker decreases to reach the steady-state levels of \(k^*\) and \(i^*\).

Capital-Labor Ratio, \(k_i\)
Changes in Population Growth

• If population growth rises, the depreciation and capital dilation line shifts up.

• The Solow model indicates that in the steady state, *higher population growth lowers the level of output per person*.
FIGURE 6.10  Response to a Rise in the Population Growth Rate (a)

Panel (a) Solow Diagram

Step 1. An increase in population growth shifts the depreciation and capital dilution line upward...

Step 2. Decreasing the capital-labor ratio.
FIGURE 6.10 Response to a Rise in the Population Growth Rate (b)

Panel (b) Movement of the Capital-Labor Ratio and Output Per Worker Over Time

Step 1. An increase in population growth causes capital and output per worker to fall...

Step 2. until capital per worker reaches its steady state level of $k^*_2$, where both capital and output per worker stop falling.
Population Growth and Real GDP Per Capita

- The Solow model predicts that higher population growth makes the average person in a country poor.
- A scatter plot of personal GDP per capita against population growth rates in nearly 100 countries provides support for this proposition.
FIGURE 6.11 International Evidence on the Relationship of Population Growth and Income Per Capita

Policy and Practice: China’s One-Child Policy and Other Policies to Limit Population Growth

• Many poor countries have implemented policies to limit population growth
• Starting in 1979, the government in China implemented a “one-child” policy, in which couples with more than one child in China were ostracized, charged penalties, and denied access to preferential housing and wage hikes.
• This policy lowered fertility rates by over 70%
Productivity Growth In The Solow Model

- To understand why economies can experience sustained increases in the standard of living over time, the Solow model need to include growth in productivity, $A$
Technology Growth and the Steady State

- Higher productivity means an upward shift of the investment function $i_t = sAk_t^{0.3}$, resulting in higher output at each level of $k_t$
- Results: The direct effect of higher productivity on output per person is amplified by the additional positive affect from a higher capital-labor ratio
FIGURE 6.12  Response to a Rise in Productivity (a)

Panel (a) Solow Diagram

Step 1. A rise in productivity increases the investment function...

Step 2. Raising the capital-labor ratio.
FIGURE 6.12 Response to a Rise in Productivity (b)

Panel (b) Movement of the Capital-Labor Ratio and the Output Per Worker Over Time

**Step 1.** A rise in productivity leads to an immediate upward jump in output per worker...

**Step 2.** and capital and output per worker rise...

**Step 3.** until capital per worker reaches its steady state level of $k_2^*$, where both capital and output per worker stop rising.
Summing Up the Solow Model

• The results of the Solow model:
  1. Economies with similar production functions and saving rates that have low (high) initial per capita income will have higher (lower) growth rates.
  2. A higher rate of saving leads to higher levels of capital and output per worker, but does not affect the long-run growth rate of these variables.
  3. Higher population growth lowers the level of output per person.
  4. Increases in productivity are amplified in generating output because there is a direct effect through the production function and an additional positive affect from a higher capital-labor ratio.
Limitations of the Solow model:

1. The model does a poor job explaining sustained increases in the standard of living.
2. Productivity growth is the only factor that explains sustained growth in standards of living, but it is an exogenous variable that is not explained by the model.