A Dynamic Model of Aggregate Demand and Aggregate Supply

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1. Introduction
   - Theoretical Background

2. Elements of Model

3. Solving the Model

4. Monetary Policy
The model emphasizes the dynamic nature of economic fluctuations.

The economy is continually exposed by various shocks.

These shocks have an immediate impact on economy’s short-run equilibrium.

The model focuses attention on how output and inflation respond over time to exogenous changes in the economic environment.

The model explicitly incorporates the response of monetary policy to economic conditions.
Output: The Demand for Goods and Services

\[ Y_t = \bar{Y}_t - \alpha(r_t - \rho) + \epsilon_t \]

- \( Y_t \): the total output of goods and services
- \( \bar{Y}_t \): the economy’s natural level of output
- \( r_t \): the real interest rate
- \( \epsilon_t \): random demand shock
- \( \alpha, \rho \): parameters greater than zero

- the key feature: the negative relationship between the real interest rate \( (r_t) \) and the demand for goods and services \( (Y_t) \)
- the parameter \( \alpha \) shows how sensitive demand is to changes in the real interest rate.
- \( \bar{Y}_t \) is economy’s natural level of output.
\( \epsilon_t \) is random variable whose values are determined by chance.
- It is zero on average but fluctuates over time.
- Animal spirits are captured by \( \epsilon_t \)
- It also captures changes in fiscal policy.

\( \rho \) is the real interest rate at which, in the absence of any shock ( \( \epsilon_t = 0 \) ) the demand for goods and services equals the natural level of output.

We can call \( \rho \) the natural level of interest.

\( \rho \) plays a key role in the setting of monetary policy.
The Real Interest Rate: The Fisher Equation

\[ r_t = i_t - E_t \pi_{t+1} \]

- \( E_t \pi_{t+1} \) is the expectation of what the inflation rate will be in period \( t+1 \) based on information available in period \( t \).
Inflation: The Phillips Curve I

\[ \pi_t = E_{t-1}\pi_t + \phi(Y_t - \bar{Y}_t) + \nu_t \]

- \(E_{t-1}\pi_t\): previously expected inflation
- \(Y_t - \bar{Y}_t\): the deviation of output from its natural level
- \(\nu_t\): exogenous supply shock

Inflation depends on expected inflation because some firms set prices in advance.
Inflation: The Phillips Curve II

- $\phi(\phi > 0)$ shows how much inflation responds when output fluctuates around its natural level.
- $\phi$ reflects both how much marginal cost responds to the state of economic activity and how quickly firms adjust prices in response to changes in cost.
- $\nu_t$ is supply shock. It is a random variable whose average value is zero.
Expected Inflation: Adaptive Expectations

\[ E_t \pi_{t+1} = \pi_t \]

- People form their expectations of inflation based on the inflation they have recently observed.
The Nominal Interest Rate: The Monetary-Policy Rule

\[ i_t = \pi_t + \rho + \theta_\pi (\pi_t - \pi_T) + \theta_Y (Y_t - \bar{Y}_t) \]

\[ r_t = i_t - \pi_t = \rho + \theta_\pi (\pi_t - \pi_T) + \theta_Y (Y_t - \bar{Y}_t) \]

for the equilibrium: \( \pi_t - \pi_T \) and \( Y_t - \bar{Y}_t \)
we get \( r_t = \rho \)
The Taylor Rule

\[ i_t = \pi_t + 2.0 + 0.5(\pi_t - 2.0) + 0.5(Y_t - \bar{Y}_t) \]
Equations

\[ Y_t = \bar{Y}_t - \alpha(r_t - \rho) + \epsilon_t \]

\[ r_t = i_t - E_t \pi_{t+1} \]

\[ \pi_t = E_{t-1} \pi_t + \phi(Y_t - \bar{Y}_t) + \nu_t \]

\[ E_t \pi_{t+1} = \pi_t \]

\[ i_t = \pi_t + \rho + \theta_\pi (\pi_t - \pi_t^T) + \theta_Y (Y_t - \bar{Y}_t) \]
The long-run equilibrium represents the normal state around which the economy fluctuates.

It occurs when there are no shocks \((\epsilon_t = \nu_t = 0)\) and inflation stabilized \((\pi_t = \pi_t - 1)\)

\[
Y_t = \bar{Y}_t \; ; \; r_t = \rho \; ; \; \pi_t = \pi_t T \; ; \; E_t \pi_{t+1} = \pi^T_t \; ; \; i_t = \rho + \pi^T_t
\]

The long-run equilibrium of this model reflects two related principles: the classical dichotomy and monetary neutrality.
The Dynamic Aggregate Supply Curve

\[ \pi_t = E_{t-1}\pi_t + \phi(Y_t - \bar{Y}_t) + \nu_t \]
The Dynamic Aggregate Demand Curve I

We begin with the demand for goods and services

\[ Y_t = \bar{Y}_t - \alpha(r_t - \rho) + \epsilon_t \]

to eliminate the endogenous variable \( r_t \), we use Fisher equation

\[ Y_t = \bar{Y}_t - \alpha(i_t - E_t\pi_{t+1} - \rho) + \epsilon_t \]

to eliminate another endogenous variable \( i_t \), we put Taylor rule and adaptive expectations

\[ Y_t = \bar{Y}_t - \alpha[\pi_t + \rho + \theta_{\pi}(\pi_t - \pi_{t}^T) + \theta_Y(Y_t - \bar{Y}_t) - \pi_t - \rho] + \epsilon_t \]
equation simplifies to

\[ Y_t = \bar{Y}_t - \alpha[\theta_\pi (\pi_t - \pi_t^T) + \theta_Y (Y_t - \bar{Y}_t)] + \epsilon_t \]

solving for \( Y_t \)

\[ Y_t = \bar{Y}_t - \left[ \frac{\alpha \theta_\pi}{(1 + \alpha \theta_Y)} \right](\pi_t - \pi_t^T) - \left[ \frac{1}{(1 + \alpha \theta_Y)} \right] \epsilon_t \]
The Dynamic Aggregate Demand Curve III

The dynamic aggregate demand curve shows a negative association between output and inflation. Its downward slope reflects monetary policy and the demand for goods and services: a high level of inflation causes the central bank to raise nominal and real interest rates, which in turn reduces the demand for goods and services. The dynamic aggregate demand curve is drawn for given values of the natural level of output $Y_n$, the inflation target $\pi_t^*$, and the demand shock $\epsilon_t$. When these exogenous variables change, the curve shifts.
Dynamic Aggregate Demand

\[ Y_t = \bar{Y}_t - \left[ \frac{\alpha\theta_{\pi}}{1 + \alpha\theta_{\gamma}} \right] (\pi_t - \pi^T_t) - \left[ \frac{1}{1 + \alpha\theta_{\gamma}} \right] \epsilon_t \]

Dynamic Aggregate Supply

\[ \pi_t = \pi_{t-1} + \phi(Y_t - \bar{Y}_t) + \nu_t \]
The Short-Run Equilibrium

The short-run equilibrium is determined by the intersection of the dynamic aggregate demand curve and the dynamic aggregate supply curve. This equilibrium determines the inflation rate and level of output that prevail in period $t$. In the equilibrium shown in this figure, the short-run equilibrium level of output $Y_t$ falls short of the economy's natural level of output $Y^*_t$. 
**Long-Run Growth**

When long-run growth causes the natural level of output $Y_t$ to increase, both the dynamic aggregate demand curve and the dynamic aggregate supply curve shift to the right by the same amount. Output $Y_t$ increases, but inflation $\pi_t$ remains the same.

1. When the natural level of output increases, ...
2. The dynamic AS curve shifts to the right, ...
3. … as does the dynamic AD curve, ...
4. … leading to growth in output ...
5. … and stable inflation.
A Shock to Aggregate Supply

A Supply Shock: A supply shock in period $t$ shifts the dynamic aggregate supply curve upward from $DAS_{t-1}$ to $DAS_t$. The dynamic aggregate demand curve is unchanged. The economy's short-run equilibrium moves from point $A$ to point $B$. Inflation rises and output falls. In the subsequent period ($t+1$), the dynamic aggregate supply curve shifts to $DAS_{t+1}$ and the economy moves to point $C$. The supply shock has returned to its normal value of zero, but inflation expectations remain high. As a result, the economy returns only gradually to its initial equilibrium, point $A$. 

1. An adverse supply shock shifts the DAS curve upward, …

2. … causing inflation to rise …

3. … and output to fall.

4. In later periods, inflation falls …

5. … and output slowly recovers.
A Shock to Aggregate Supply

(a) Supply Shock
(b) Output
(c) Real Interest Rate
(d) Inflation
(e) Nominal Interest Rate

The Dynamic Response to a Supply Shock: This figure shows the responses of the key variables over time to a one-time supply shock.
A Shock to Aggregate Demand

A Demand Shock: This figure shows the effects of a positive demand shock in period t that lasts for five periods. The shock immediately shifts the dynamic aggregate demand curve to the right from $DAD_{t-1}$ to $DAD_t$. The economy moves from point A to point B. Both inflation and output rise. In the next period, the dynamic aggregate supply curve shifts to $DAS_{t+1}$ because of increased expected inflation. The economy moves from point B to point C, and then in subsequent periods to points D, E, and F. When the demand shock disappears after five periods, the dynamic aggregate demand curve shifts back to its initial position, and the economy moves from point F to point G. Output falls below its natural level, and inflation starts to fall. Over time, the dynamic aggregate supply curve starts shifting downward, and the economy gradually returns to its initial equilibrium, point A.
A Shock to Aggregate Demand

The Dynamic Response to a Demand Shock: This figure shows the responses of the key variables over time to a positive 1-percent demand shock that lasts for five periods.

1. **Demand Shock**
   - Time: t-2, t, t+1, t+2, t+3, t+4, t+5, t+6, t+7, t+8, t+9, t+10, t+12

2. **Output**
   - Y_t: 99.0, 100.0, 100.5, 101.0

3. **Inflation**
   - π_t: 0.0, 0.5, 1.0, 1.5, 2.0, 2.5, 3.0

4. **Real Interest Rate**
   - r_t: 1.0, 1.5, 2.0, 2.5, 3.0

5. **Nominal Interest Rate**
   - i_t: 2.0, 3.0, 4.0, 5.0, 6.0
A Shift in Monetary Policy

A Reduction in Target Inflation
A permanent reduction in target inflation in period $t$ shifts the dynamic aggregate demand curve to the left from $DAD_{t-1}$ to $DAD_t$, where it then stays. Initially, the economy moves from point A to point B. Both inflation and output fall. In the subsequent period, because expected inflation falls, the dynamic aggregate supply curve shifts downward. The economy moves from point B to point C in period $t+1$. Over time, as expected inflation falls and the dynamic aggregate supply curve repeatedly shifts downward, the economy approaches a new equilibrium at point Z. Output returns to its natural level $Y^*_t$, and inflation ends at its new, lower target (1 percent).
A Shift in Monetary Policy

(a) Inflation Target

The Dynamic Response to a Reduction in Target Inflation. This figure shows the responses of the key variables over time to a permanent reduction in the target rate of inflation.

(b) Output

(c) Real Interest Rate

(d) Inflation

(e) Nominal Interest Rate
The Tradeoff Between Output and Inflation

\[ Y_t = \bar{Y}_t - \left[ \frac{\alpha \theta \pi}{(1 + \alpha \theta_Y)} \right] (\pi_t - \pi_T) - \left[ \frac{1}{(1 + \alpha \theta_Y)} \right] \epsilon_t \]
References

- Mankiw, Macroeconomics, 9th Edition - Chapter 15